Testing the Efficiency of Emerging Markets: the Case of the Baltic States

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There is little evidence on the efficiency of the early stage of the capital market in transition countries, although market structure developments and the learning process could define the framework for efficient markets. The article tries to find out whether financial markets are efficient in the three Baltic States and if not, whether there are any signs of evolving to the efficient capital market. To answer these questions the analysis combines the methodology for testing the efficiency of capital market using the variance ratio robust to heteroscedasticity with the state-space representation, which enables us to use an efficient filtering technique – the Kalman filter – to get time varying autocorrelations. The official Estonian, Latvian, and Lithuanian stock exchange market indices TALSE, DJRSE, and LITIN comprising the most liquid parts of the stock market in a respective country are analysed. The main conclusion to be drawn from the analysis is that financial markets in the Baltic States are, with some turbulence, approaching weak form of efficiency.

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Introduction

The question of the financial market efficiency is crucial for transition economies, as it determines the possibility to diversify non-systemic risks of portfolios to forecast stock prices and returns. In general, efficiency will determine the financial market capacity to allocate free capital efficiently (see Fama (1970, Grossman (1989), Stiglitz (1981)). This article explores whether financial markets in the three Baltic States are efficient.

The major original contribution of this article is the analysis of efficiency during transition process. To be more precise, we specify the requirement for a statistic to be suitable to evaluate the information efficiency state and its changes in a transition economy's capital market and propose a statistic, which is consistent with these required properties.

Our general view is that the financial market has gradually moved towards efficiency as the Baltic States were going through the transition process. As pointed out in other papers on transition economies (see Hall (1993), Hall and O'Sullivan (1994), Hall and Koparanova (1995), Greenslade and Hall (1996), Ghysels and Cherkaou (1999), Basdevant (2000)), econometric modelling can still be a useful tool, but it must take an explicit account of the form of change that has taken place. In the theoretical literature on financial markets (see Campbell, Lo, and MacKinlay (1997), Malkiel (1989)) efficiency is defined rather as a static characteristic and the alternative is efficient market versus inefficient or relatively inefficient. Therefore, it is not surprising that capital markets of transition economies were not even found weakly efficient with frequent data (Emerson *et al* (1996), Macskasi (1996)). This especially holds in the early transition stages when institutional and informational market structure is poorly defined[1]. Nevertheless, the economic system transformation encourages financial relationships and markets to change. Hence, to better understand economic processes in transition economies, one should focus on understanding whether transition also leads to an improvement in market efficiency. Using standard econometric techniques would lead to test efficiency versus inefficiency on the whole sample and; therefore, the result might be biased by the structural change happening at the beginning of the period. This is why it is expedient to use other techniques that allow integrating the structural change more explicitly. In this article, we apply time varying variance ratios estimated by the Kalman filter technique. The capital markets of Estonia and Lithuania are shown to move towards efficiency with some turbulence and the Latvian capital market structure does not seem to be satisfactory for the establishment of weak form efficiency.

Despite the fact that markets are globally incomplete, it is possible to identify informational efficient financial markets making an additional assumption on the equilibrium price formation model[2]. On a market in which prices always *fully reflect* available information is called *efficient* (see Fama (1991)), and finance theory usually distinguishes three types of efficiency: weak, semi-strong and strong, depending on the assumptions made on the

[1] This fact is consistent also with other emerging markets behaviour (Harvey (1994), Callen (1991)). The inefficiency could be extremely influenced by uncertainty and liquidity. Most likely, the liquidity constraint is the reason for the emergence of efficiency during Asian-Russian crises.

[2] Efficiency testing is always related to testing the joint hypothesis on the efficiency and a particular equilibrium asset price return model (Fama (1991), LeRoy (1989), Malkiel (1989)). Consequently, the rejection of the efficiency hypothesis could be due to deficiency in the pricing model and this is the common problem for all market efficiency tests.

information set available[3].

- Weak efficiency holds if *information contained in former prices* is incorporated into market prices.
- Semi-strong efficiency holds if publicly *available information set* is incorporated.
- Strong form efficiency holds if *all information known to any market participant* is incorporated into market prices.

The model we adopted is aimed at testing weak efficiency versus inefficiency. We limit the analysis to the weak form, because the financial market in the Baltic States – just like in other transition economies – is much less developed than that in western economies and we could hardly expect it to be even semi-strongly efficient, especially during its early development stage. Due to the short sample period, only daily data are analysed. We took DJRSE (for Latvia), LITN (for Lithuania), and TALSE (for Estonia) indices that are daily capital weighted indices of price movements (for more details see the Annex).

[3] According to a basic general equilibrium approach, any price on any market should summarise all the information needed for each participant to make a decision. When markets are incomplete, some information is not available and may bias the decision made by the participants. One of the greatest arguments to reject the assumption of efficient markets is that future markets and future prices are unknown. Hence, agents have to forecast future prices, knowing current and/or past prices. The problem is then to define the actual set of information relevant to forecast prices.

1. Methodology for Testing Efficiency

In this Section, the methodology used for testing the efficiency is first described in short. Further, we specify the requirements to the statistic for evaluating the changing efficiency and propose a time varying variance ratio. It is based on time-varying autocorrelation coefficients and enables the model to capture a structural change that has occurred. The coefficients are estimated by the Kalman filter[4]. Those points are discussed further below, and the results are presented in Section 2.

1.1. Martingale Hypothesis: the Null Hypothesis

To denote an asset price[5] at the moment *t* by p_t, let the weak form efficiency information set, known by market participants, at date *t–1* be

(1) $$I_{t-1} = \{p_{t-1}, p_{t-2}, ...\}.$$

If a market exhibits the weak form efficiency, the price has the property

(2) $$\mathrm{E}(p_t \mid I_{t-1}) = p_{t-1},$$

ie it is a martingale (see LeRoy S.F. (1989)). Hence, the price difference[6] ε_t

(3) $$\varepsilon_t = p_t - p_{t-1}$$

is a martingale difference given information in the set I_{t-1}. However, as mentioned earlier, there is no reason why the efficiency should hold during the first stage of transition, while it is likely that markets become more efficient as time goes. We therefore define the time varying autocorrelation

(4) $$E(\varepsilon_t, \varepsilon_{t-j}) = \rho_{t,k}$$

and for a particular moment t we test the null hypothesis

(5) $$H_0: \quad \forall j \neq 0 \quad \rho_{t,k} = 0.$$

We reject the null, when some significant auto-correlation is found

(6) $$H_1: \quad \exists j \neq 0 \quad \rho_{t,k} \neq 0.$$

Next, we discuss our proposal to use a time varying variance ratio statistic for heteroscedasticity consistent inference and a respective state-space model to get relevant estimates of autocorrelation coefficients.

1.2. Testing the Null Hypothesis

1.2.1. Desired Properties of the Test Statistic

In order to assess properly whether an emerging market moves towards efficiency, we would like the statistic to have these properties:

- The magnitude of auto-correlation should be allowed to change over time.
- The structure of auto-correlation should be allowed to change over time.
- The statistic must be able to take into account heteroscedasticity.

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[4] For other testing procedures and market efficiency theoretical aspects see LeRoy (1989), Bollerslev and Hodrick (1992), Mills (1999, Ch.2 and Ch.4), Campbell, Lo, and MacKinlay (1997, Ch.2), Beechey (2000).

[5] Here the index value logarithm stands as a proxy for an asset basket price. We use price logarithm to avoid common problems related to theoretical non-positive price possibility when it is normally distributed (Campbell (1997), Ch.1) and non-pure martingale behaviour, only if the price without dividend is used instead of the rate of returns (LeRoy (1989)).

[6] Possibly adjusted for a mean.

- The structure of heteroscedasticity should be general, as it would be inconsistent to assume a fixed structure of heteroscedasticity, while the auto-correlation structure is allowed to change over time.

Therefore, we first require the statistic to measure the changing autocorrelation magnitude for a particular lag, ie a time varying autocorrelation measure should be used. For a possible solution using the state-space framework and Kalman filtering as well as its application in practice see Emerson *et al* (1996), Rockinger and Urga (2001), Zalewska and Hall (1999). Nevertheless, the efficiency dynamics here was evaluated by the graphical inspection of only the first lag autocorrelation changes. However, this is not sufficient, because it only measures changes in a particular autocorrelation, while in an emerging market the correlation structure may also change significantly with time. For instance, when the autocorrelation of higher order is significant and does not diminish and the first-order autocorrelation coefficient does, the inspection of only the first-order autocorrelation would mislead to an inference on reaching the state of efficiency, and vice versa (see Simulation Results in Subsection 1.2.4). Consequently, inferences based only on the first-order time-varying autocorrelation parameter estimate without additional arguments should be drawn carefully.

Therefore, the second requirement states a necessity to infer about the efficiency dynamics as a whole, ie to comprehend the autocorrelations of different lags with one statistic. This might not be so important for the study of a more developed capital market efficiency, because in a nearly efficient market new information should be incorporated as soon as possible and the first lag value is likely to account for – and adjust to – a huge part of new information. In the market of a transition country this might not be a reasonable assumption.

If ε_t were from a fixed distribution with a finite variance, the joint null hypothesis on the non-significance of different lag order autocorrelations could be tested using Ljung-Box statistic (Bollerslev and Hodrick (1992), Mills (1999), Campbell, Lo, and MacKinlay (1997, Ch.2)), with the only difference that the time-varying autocorrelation estimates should be used to satisfy the first requirement. However, heteroscedasticity is a common feature of financial time series and this may result in incorrect conclusions. Therefore, the statistic should be able to account for it.

It is possible to model an explicit heteroscedasticity structure in a GARCH framework (see Bollerslev (1986), French *et al* (1987), Emerson *et al* (1996), Rockinger and Urga (2001)). However, there is no reason to assume the heteroscedasticity structure to be specific, ie time invariant, while all the circumstances in a developing market are assumed to change. This disqualifies the GARCH modelling for this particular case.

1.2.2. Time-varying Variance Ratios

To evaluate the efficiency state of the capital market in a transition economy, we suggest using the time-varying variance ratio, which allows getting a heteroscedasticity robust inference without specifying certain structure of conditional heteroscedasticity, and enables us to evaluate different structures of autocorrelation.

To be more precise, we employ the variance ratio test (for a brief variance ratio description, important to our analysis and coupled with relevant references see the Annex) as a basis and define the time varying variance ratio (TVR) as follows:

(7) $$V_{pt} = 1 + 2\sum_{k=1}^{p}\left(1 - \frac{k}{p+1}\right)\rho_t^k ,$$

where p is the maximum time period of correlated price increments and ρ_t^k is a k lag autocorrelation at time t.

To estimate a time-varying autocorrelation ρ_t^k for each k of the process described in equation (3), we set up p state-space models as follows[7] for each value of $k \in \{1..p\}$:

(8) $$\varepsilon_t = \rho_t^k \varepsilon_{t-k} + \nu_t^k$$

(9) $$\rho_t^k = \rho_{t-1}^k + \eta_t^k ,$$

with $\forall k\ \nu^k{}_t \sim nid(0, \sigma^2_{\nu k}),\ \eta^k{}_t \sim nid(0, \sigma^2_{\eta k})$,and $\forall k,j\ E(\nu^k{}_t, \eta^k{}_{t-j})=0$. The set of equations (8) and (9) constitutes the state-space form that may be directly estimated by a Kalman filter (see the Annex)[8].

After getting a vector estimate of V_{pt}, we define the variance ratio statistic M_{pt}, robust to heteroscedasticity

(10) $$M_{pt} = \left(\hat{V}_{pt} - 1\right)\left(4\sum_{k=1}^{p}\left(1 - \tfrac{k}{p+1}\right)^2 \left(\sum_{t=k+1}^{T} \varepsilon_t^2 \varepsilon_{t-k}^2 \Big/ \left(\sum_{t=1}^{T} \varepsilon_t^2\right)^2\right)\right)^{-\frac{1}{2}} ,$$

which is asymptotically standard normally distributed and is consistent with the required properties described earlier. Under the null hypothesis (5) the following holds: $\forall(k,t)\ \rho_t^k = 0$ and respectively $V_{pt}=1$ and asymptotically $M_{pt}=0$.

However, one relevant question remains open – how to choose the maximum autocorrelation lag order p? If some specific price-generating model was specified, the respective autocorrelation structure might be derived. However, we do not test for a specific alternative hypothesis and just test the null against a broad range of alternative hypothesis by taking several different lag orders to evaluate the respective price increment autocorrelations. The minimum p=1 and maximum p=7 orders for test statistics comprising one and seven autocorrelation lags were chosen because of inspection of the price increment autocorrelation function.

1.2.3. Some Properties of the Proposed Statistic

When dealing with real data we do not know the true data generating process (DGP), therefore, some behavioural properties of the proposed statistic are described. These are mainly predetermined by the autocorrelation function (ACF) behaviour of DGP. We utilise the properties of stationary ARMA processes (for their ACF description see Hamilton (1994)).

If data are generated by a moving average (MA) process, the time varying variance ratio statistics (TVRS) of different order should not differ a great deal after the true order of DGP has been reached, because the ACF of MA process drops to zero after the true MA order is

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[7] Data are adjusted with respect to the mean ($\varepsilon_t = lnP_t - lnP_{t-1} - \mu_{\Delta lnP}$, where P_t is the index value at date t and $\mu_{\Delta lnP}$ is an estimate of unconditional mean of price logarithms changes).

[8] Having the entire data sample for the construction of VR, we use the smoothed estimates (see *ibid*), ie we estimate $\hat{\rho}_{t|T}^k = E\left(\rho_t^k \middle| \varepsilon_1 ... \varepsilon_T\right)$.

reached. However, they differ slightly, because increasing p will change the weights attached to the same values of the time-varying correlation estimates. If data are generated by an autoregressive process (AR), the different order TVRS may considerably differ from each other because the ACF of AR does not drop to zero, but decays as a mixture of dumped processes. The relationship between TVRS for different p will be even more complicated because of changing weights. Nevertheless, the difference should disappear when the order of p differs much from the real order of the process. For a more general ARMA process, the described AR properties hold when p is greater than the order of MA, and prior to that the behaviour of the test statistic is more complicated than for a separate process.

Note that, under specific circumstances, TVRS does not completely satisfy our second requirement, ie to comprehend the autocorrelations of different lags with one statistic and to be robust to different autocorrelation structures. A higher order TVRS might become not significant in the presence of several separate significant autocorrelations with different signs. This is because the positive and negative autocorrelations of a more complex process might counteract one another. Therefore, the Ljung-Box statistic with squared autocorrelations robustified to heteroscedasticity might be preferred. Negative autocorrelation is less expectable in our case, and we employ the TVRS to estimate the dynamics of efficiency. However, to assess the state of efficiency we propose not to rely on a particular lag order p statistic, but to test it for different p. In case any of them are significant, the null hypothesis is rejected. Investigation of several statistics has also some advantages, because different trends of TVRS may reveal the changing autocorrelation structure.

1.2.4. Simulation Results

In order to get an understanding of time varying variance ratio capability to reflect the changing parameters and to cope with the changing autocorrelation structure, we present the results of simple simulation. Only the homoscedastic processes are simulated.

First of all, from 1000 values of normally and identically distributed random errors[9] ε_t~nid(0,0.00023) we simulated the dynamics of the MA(1) and AR(1) with time varying parameters. For comparison, several estimated different order M_{pt} statistics are presented (see Figures 1 and 2).

[9] For variance we use the whole sample estimated variance value of $r_t = log(LITIN_t) - log(LITIN_{t-1}) - \mu_{\Delta lnP.}$

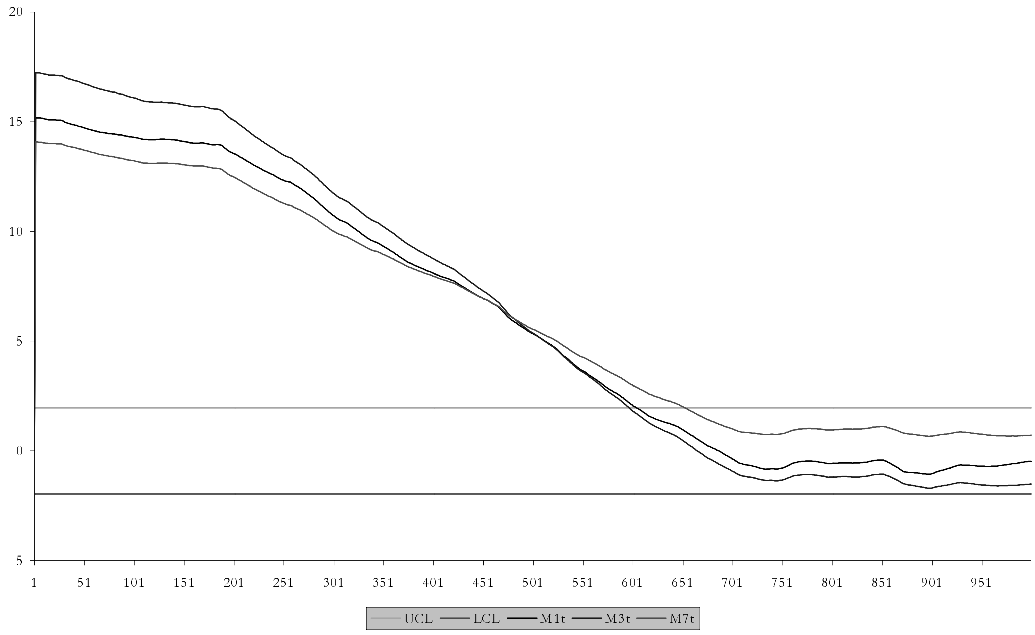

Figure 1. Test of the statistic M_{pt} suitability for a time-varying parameter AR(1)[10]

Model: $\varepsilon_t=\rho_t\varepsilon_{t-1}+v_t$, v_t~nid(0, 0.00023), ε_0=0, $\rho_t=\begin{cases} 0.5, t \le 200 \\ 0.5\left(1-\dfrac{t-200}{500}\right), 201 \le t < 700 \\ 0, t \ge 701 \end{cases}$, t=1,...,1000

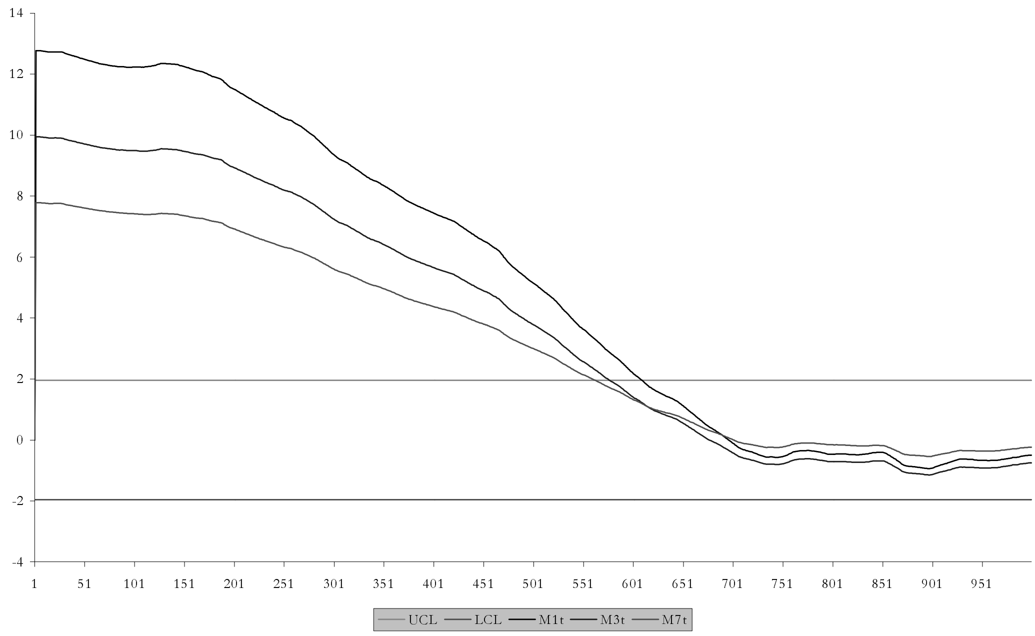

Figure 2. Test of the statistic M_{pt} suitability for a time-varying parameter MA(1)

Model: $\varepsilon_t= v_t+\rho_t v_{t-1}$, v_t~nid(0, 0.00023), ε_0=0, $\rho_t=\begin{cases} 0.5, t \le 200 \\ 0.5\left(1-\dfrac{t-200}{500}\right), 201 \le t < 700 \\ 0, t \ge 701 \end{cases}$, t=1,...,1000

[10] The 95% upper and lower confidence levels are denoted as UCL and LCL, respectively.

The TVRS of different order of *p* behaves according to earlier described properties.

The ability of time varying variance ratio statistic to reflect a changing correlation structure is presented by simulating MA(2) and AR(2) processes with time varying parameters. The first parameter value is decreasing and the second parameter value for a particular observation just increases as much as the first parameter value decreases. Thus, if we did not regard the fact that sooner incorporation of information could be viewed as a sign of improving efficiency, the whole impact of two parameters would always result in a constant inefficiency measure. The respective test statistics for AR(2) and MA(2) processes are presented in Figure 3 and 4, respectively.

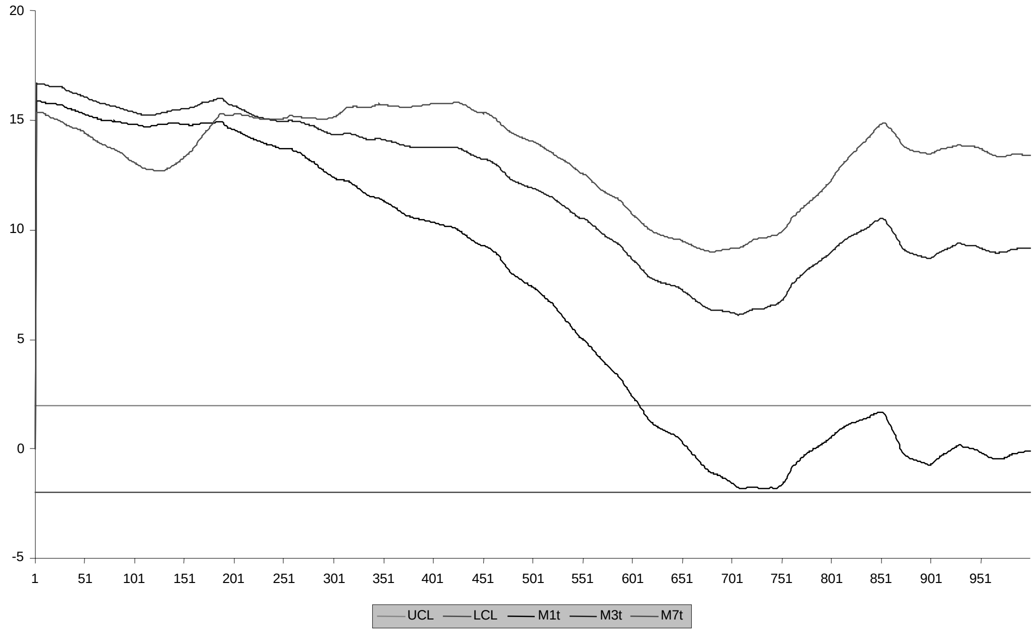

Figure 3. Test of the statistic M_{pt} suitability for a time-varying parameters AR(2)

Model: $\varepsilon_t=\rho_{1t}\varepsilon_{t-1}+(0.5-\rho_{1t})\varepsilon_{t-2}+v_t$, v_t~nid(0, 0.00023), ε_0=0, $\rho_{1t}=\begin{cases} 0.5, t \le 200 \\ 0.5\left(1-\dfrac{t-200}{500}\right), 201 \le t < 700 \\ 0, t \ge 701 \end{cases}$, t=1,...,1000

Figure 4. Test of the statistic M_{pt} suitability for a time-varying parameters MA(2)

Model: $\varepsilon_t = v_t + \rho_{1t} v_{t-1} + (0.5 - \rho_{1t}) v_{t-2}$, $v_t \sim \text{nid}(0, 0.00023)$, $\varepsilon_0 = 0$, $\rho_{1t} = \begin{cases} 0.5, t \le 200 \\ 0.5\left(1 - \dfrac{t-200}{500}\right), 201 \le t < 700 \\ 0, t \ge 701 \end{cases}$, t=1,...,1000

Due to different weightings in different order VR, the M_{3t} and M_{7t} statistics slightly differ, however the difference will diminish with an increasing *p* value. The TVRS again captures the dynamics of the parameters and higher order statistics clearly indicate significant autocorrelations. However, if only first order autocorrelation were analysed, the conclusion would be misleading.

2. Dynamics of Efficiency

In this Section, the dynamics of the efficiency of the three capital markets in the Baltic States is described. The autocorrelations are calculated from the returns of Estonian TALSE, Latvian DJRSE, and Lithuanian LITIN indices. The methodology described in Subsection 1.2 is implemented, and the robust to heteroscedasticity time-varying statistics M_1, M_3, and M_7 with 95 per cent confidence level for the null hypothesis are plotted in Figure 5, 6, and 7.

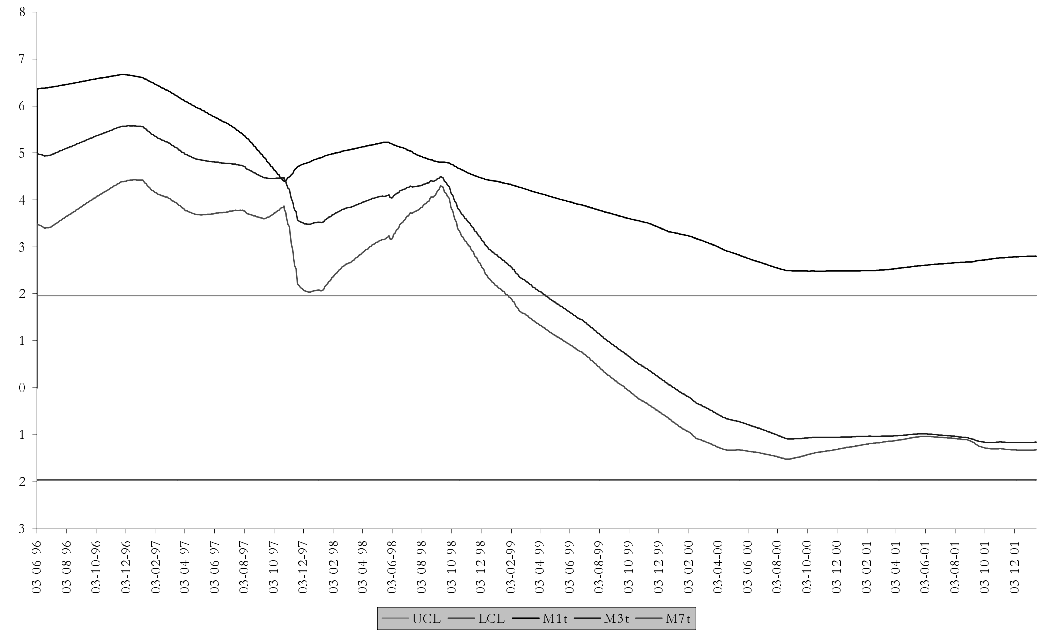

Figure 5. Dynamics of the standardised time-varying heteroscedasticity robustified variance ratio statistic M_{pt} for TALSE returns

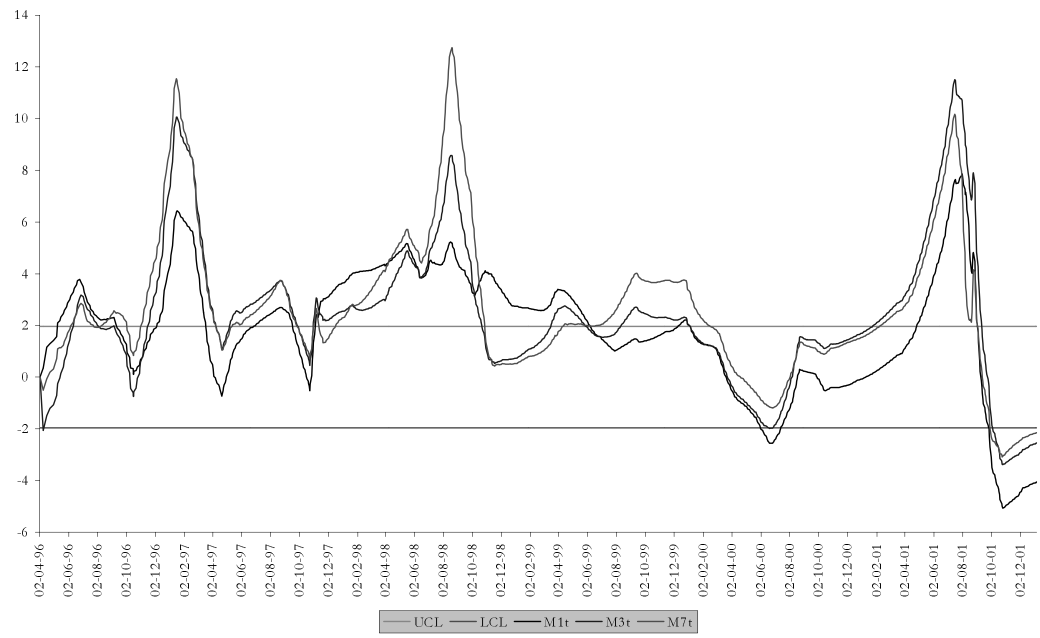

Figure 6 Dynamics of the standardised time-varying heteroscedasticity robustified variance ratio statistic M_{pt} for DJRSE returns

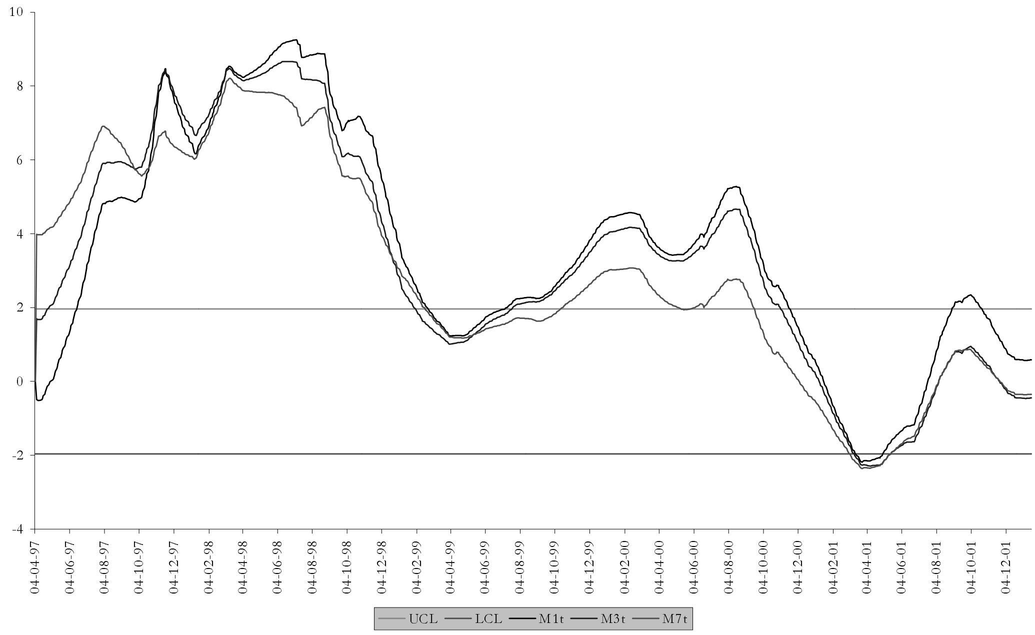

Figure 7. Dynamics of the standardised time-varying heteroscedasticity robustified variance ratio statistic M_{pt} for LITIN returns

2.1. Estonia

As we have expected, the most inefficient state occurred at the beginning of the analysed period. A clear and relatively steady tendency of its diminishing can be seen in all the statistics. With an increase in order of *p*, the autocorrelations even become negative and stipulate insignificant TVRS of higher order (for autocorrelations, estimated at different periods, as well as static variance ratio robust to heteroscedasticity, calculated in a "moving windows" framework see the Annex). The first order statistic, however, is significant and; therefore, some inefficiency persists. Nevertheless, the drop in inefficiency is huge and completely consistent with our expectations.

2.2. Latvia

In case of Latvia, there is no clearly expressed tendency for diminishing inefficiency. If the period until the last peak of inefficiency is analysed, then some not very well defined tendencies might be discovered. However, due to extremely huge inefficiency even in the very last period there are still possibilities to predict future price movements in the Latvian market in particular periods. It is interesting to note that a look at the three highest peaks reveals that the higher order autocorrelations are clearly important and might be useful for price movement prediction.

2.3. Lithuania

The capital market in Lithuania is clearly approaching the weak-form efficiency, although not as steadily as the Estonian one, but rather with some repeating smaller turbulences. The last of them is actually not significant. Therefore, the case of Lithuania is consistent with our expectations of improving market conditions and the efficiency state as well. We also note that the different order TVRS dynamics for LITIN is very similar indicating no significant change in the structure of autocorrelation.

3. Conclusion

Standard techniques are not fit to test the weak-form capital market efficiency of a transition country and the testing statistic should satisfy some special requirements. Combining the variance ratio methodology with the state-space framework, we suggest using the time-varying variance ratio statistic robust to heteroscedasticity based on time-varying autocorrelations, which are estimated using the Kalman filter technique. This enables us to monitor the trajectory of the efficiency state without specifying the particular heteroscedasticity structure. Countersign autocorrelations might distort the picture though, and to avoid this the time-varying heteroscedasticity robustified Ljung-Box statistic might be an extension to consider for future research.

We noticed a clearly expressed motion to the weak-form efficiency in the Estonian and Lithuanian capital markets. Although there is yet relatively small inefficiency in these two markets (especially in the Estonian one), a slight autocorrelation observed in the frequent data could be explained by standard reasons, such as transaction costs, expenses for acquiring information, etc (Grossman and Stiglitz (1980)). In the Latvian market, we have found a huge inefficiency even at the very end of the analysed period, possibly indicating that the capital market structure is not developed enough to ensure even the nearly weak-form efficiency.

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Annex

Variance ratio test

The variance ratio test makes use of the fact that the variance of a random walk process increases linearly with time, so that the q period variance of the iid residuals is equal to q times the variance of it. Even without restricting to a random walk, assuming p_t to have one unit root and using the stationarity of ε_t, the variance ratio could be expressed, as showed by Cochrane (1988), as a function of autocorrelation coefficients

(11) $$V_q = \frac{Var(\varepsilon_t + ... + \varepsilon_{t-q+1})}{qVar(\varepsilon_t)} = 1 + 2\sum_{k=1}^{q-1}\left(1 - \frac{k}{q}\right)\rho_k ,$$

where ρ_k is a k^{th} order autocorrelation coefficient of ε_t. If the process is a pure random walk, the autocorrelations are zeros and V_q=1.

In our case, we do not assume the returns ε_t to be iid, although even in the presence of conditional heteroscedasticity under certain assumptions (see Lo and MacKinlay (1988,1989) or Campbell, Lo, and MacKinlay (1997, p.54)), implying the asymptotically uncorrelated ε_t, the variance ratio must still approach to one as the number of observations increases. For a statistical inference that V_q=1, we use the robustified test statistic, proposed by Lo and MacKinlay (1988).

(12) $$M_q = \left(\hat{V}_q - 1\right)\left(4\sum_{k=1}^{q-1}\left(1 - \tfrac{k}{q}\right)^2\left(\sum_{t=k+1}^{T}\varepsilon_t^2\varepsilon_{t-k}^2 \Big/ \left(\sum_{t=1}^{T}\varepsilon_t^2\right)^2\right)\right)^{-\frac{1}{2}}$$

where $\hat{V}_q$ is an empirical analogue of V_q in (11). (12) is a special case for the process in (3) and its general expression can be found in Lo and MacKinlay (1988, 1989) or Campbell, Lo, and MacKinlay (1997, p 55). Despite the presence of general heteroscedasticity under the null hypothesis of random walk the robustified variance ratio statistic is asymptotically normally distributed $M_q \overset{a}{\sim} N(0,1)$, so that we test in the usual way, whether the value of the statistic in (12) differs significantly from zero.

Also note that, in the text, we use a slightly changed notation, i.e., instead of q we use $p+1$. This is because, in our case, the autocorrelation order $p=q-1$ is important, but not the q period difference.

The state space form and the Kalman filter

This section illustrates how the Kalman filter (see Kalman (1960) and (1963)) is implemented. A more detailed presentation can be found in Gourieroux and Monfort (1995), Hamilton (1994), Harvey (1987, 1989), Hall *et al* (1992), or Hall (1993).

Let

(13) $$Y_t = Z_t A_t + \varepsilon_t$$

be the measurement equation, where Y_t is the vector of measured variables, A_t is the state vector of unobserved variables, Z_t is the matrix of parameters and $\varepsilon_t \sim N(0, H_t)$. The state equation is then given as follows

(14) $$A_t = T_t A_{t-1} + \eta_t$$

where T_t is the matrix of parameters and $\eta_t \sim N(0,Q_t)$. The disturbances ε_t and η_t are assumed to be uncorrelated at all lags.

Let a_t be the optimal estimator of A_t based on the observations up to and including y_t, and $a_{t|t-1}$ is the estimator based on the information available in t-1. We can define that

(15) $$P_{t-1} = E\left((A_{t-1} - a_{t-1})(A_{t-1} - a_{t-1})' \right)$$

Given a_{t-1} and P_{t-1}, the optimal estimator of A_t is:

(16) $$a_{t|t-1} = T_t a_{t-1}$$

while the covariance matrix of the estimator is:

(17) $$\begin{aligned} P_{t|t-1} &= E\left((A_t - a_{t|t-1})(A_t - a_{t|t-1})' \right) \\ &= T_t P_{t-1} T_t' + Q_t \end{aligned}$$

When Y_t is known, the estimator can be updated:

(18) $$a_t = a_{t|t-1} + P_{t|t-1} Z_t' \left(Z_t P_{t|t-1} Z_t' + H_t \right)^{-1} \left(Y_t - Z_t a_{t|t-1} \right)$$

(19) $$P_t = P_{t|t-1} - P_{t|t-1} Z_t' \left(Z_t P_{t|t-1} Z_t' + H_t \right)^{-1} Z_t P_{t|t-1}$$

Equations (16) to (19) represent then the Kalman filter equations.

In the Gaussian model, the Kalman filter yields the conditional mean and covariance matrix of distribution of A_t given the information available at time t. Thus

(20) $$a_t = E(A_t \mid Y_t)$$

(21) $$P_t = E\left([A_t - E_t(A_t)][A_t - E_t(A_t)]' \right)$$

The conditional mean is the least minimum mean square estimate of A_t. The expression of this estimator applies to any set of observations. This estimator minimises the mean square errors when the expectation is taken over all the variables in the information set rather than being conditional on a particular set of values (see Anderson and Moore (1979) or Harvey (1989) for a detailed discussion). Thus the conditional mean estimator, a_t, is the minimum mean square estimator of A_t. This estimator is unconditionally unbiased and the unconditional covariance matrix of the estimator is the P_t matrix given by the Kalman filter. Proofs of these results can be found in Anderson and Moore (1979), Ducan and Horn (1972) or Harvey (1981).

Until now, we assumed that the matrices Z_t, T_t, H_t, Q_t are known. In general they may depend on an unknown parameter vector ψ, which can be estimated by maximum likelihood provided sufficient regularity conditions are satisfied.

The Kalman filter results in the estimator a_t that gives the minimum mean square estimate of A_t having the data sample up to t (see (20)). However, having the entire data sample, we are interested in the smoothed estimate

(22) $a_{t|T} = E(A_t \mid Y_T)$.

The fixed interval smoothed estimates are calculated working backwards from the last value of the earlier estimate $a_{T/T}=a_T$, $P_{T/T}=P_T$ as

(23) $a_{t|T} = a_t + P_t^*(a_{t+1/T} + T_{t+1}a_t)$.

(24) $P_{t|T} = P_t + P_t^*(P_{t+1/T} + P_{t+1/t})P_t^{*\prime}$,

where $P_t^* = P_t T_{t+1}' P_{t+1/t}^{-1}$, t=T-1,...,1.

The principal characteristic of the time series model is that the observations are dependent. Hence, the standard formula is not applicable to compute the likelihood function. Instead, the definition of a conditional probability density function is used to write the joint density function (see Crowder (1976) and Schweppe (1965))

(25)
$$\begin{aligned} LogL = &-\frac{N(T-k)}{2}Log(2\pi) - \frac{1}{2}\sum_{t=k}^{T} Log\left|Z_t P_{t|t-1} Z_t' + H_t\right| \\ &-\frac{1}{2}\sum_{t=k}^{T} \nu_t' \left(Z_t P_{t|t-1} Z_t' + H_t\right)^{-1} \nu_t \end{aligned}$$

where N is the number of elements contained in the vector Y_t, k is the number of periods needed to derive estimates of the state vector and with ν_t defined as follows

(26)
$$\nu_t = Y_t - Z_t a_{t|t-1}.$$

The vector ν_t can be interpreted as the vector of prediction errors since the conditional mean is also the minimum mean square estimator of Y_t. Hence, the likelihood function can be expressed as a function of the one-step-ahead prediction errors, weighted appropriately.

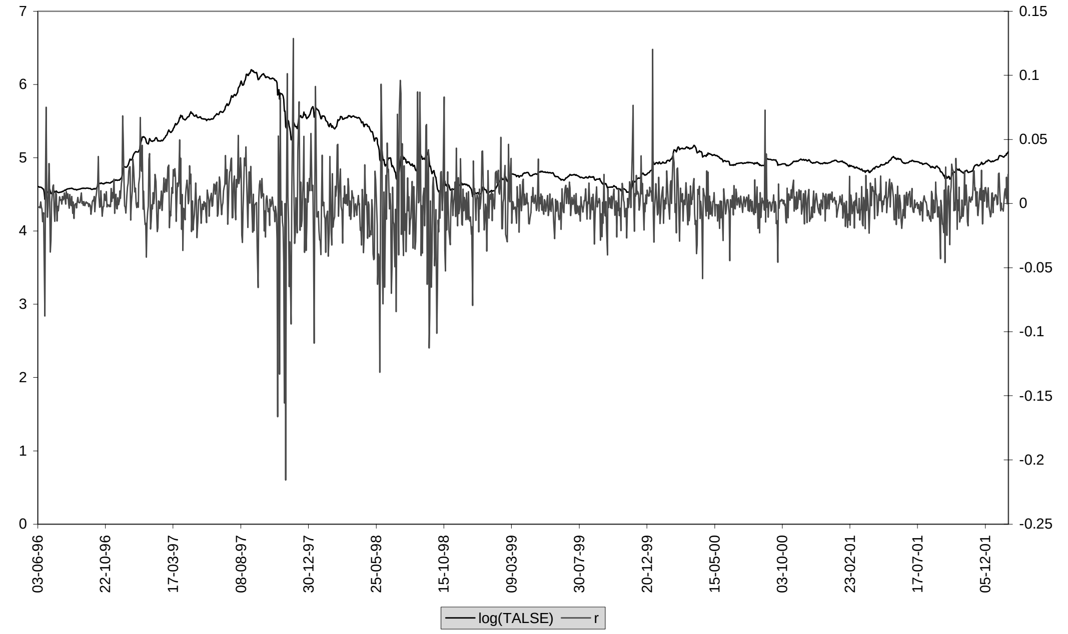
Description of DJRSE, LITIN, and TALSE indices

Country	Index	Index type	Starting value	Starting date	Internet source
Estonia	TALSE	Capitalisation weighted	100	03.06.96	http://www.tse.ee/english/general/general.php?lk=overview/default.html%23Talse http://www.esm.ee/english/
Latvia	DJRSE	Capitalisation weighted	100	02.04.96	http://www.rfb.lv http://www.rfb.lv/info/dowjones.html
Lithuania	LITIN	Capitalisation weighted	1000	07.04.97	www.nse.lt

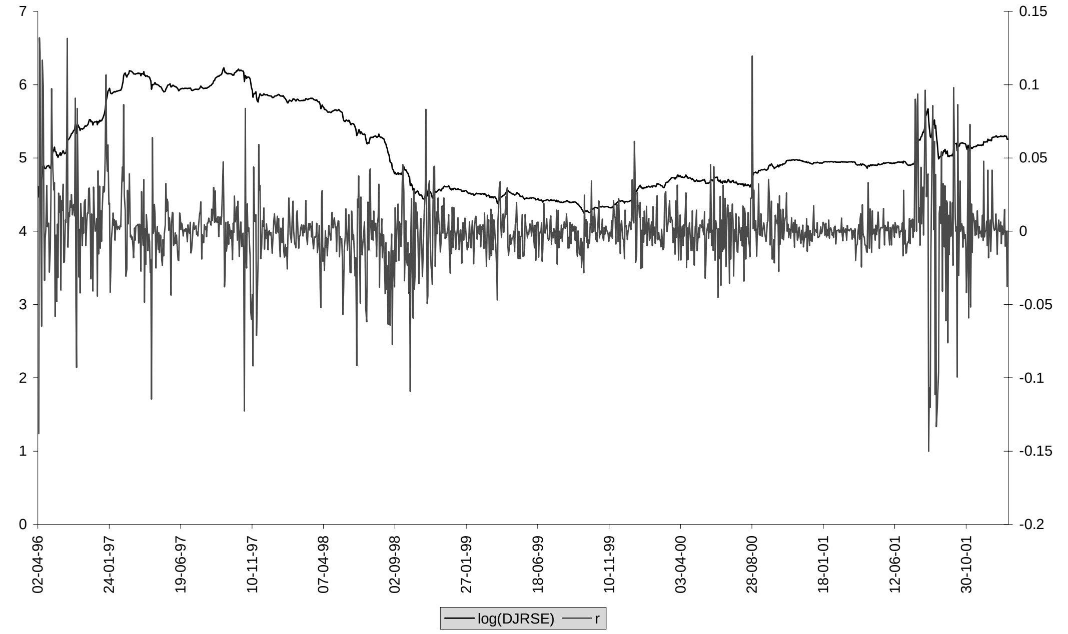
Plots of indices and their returns

Recall that $r_t = log(INDEX_t) - log(INDEX_{t-1}) - \mu_{\Delta lnP} = \varepsilon_t$.

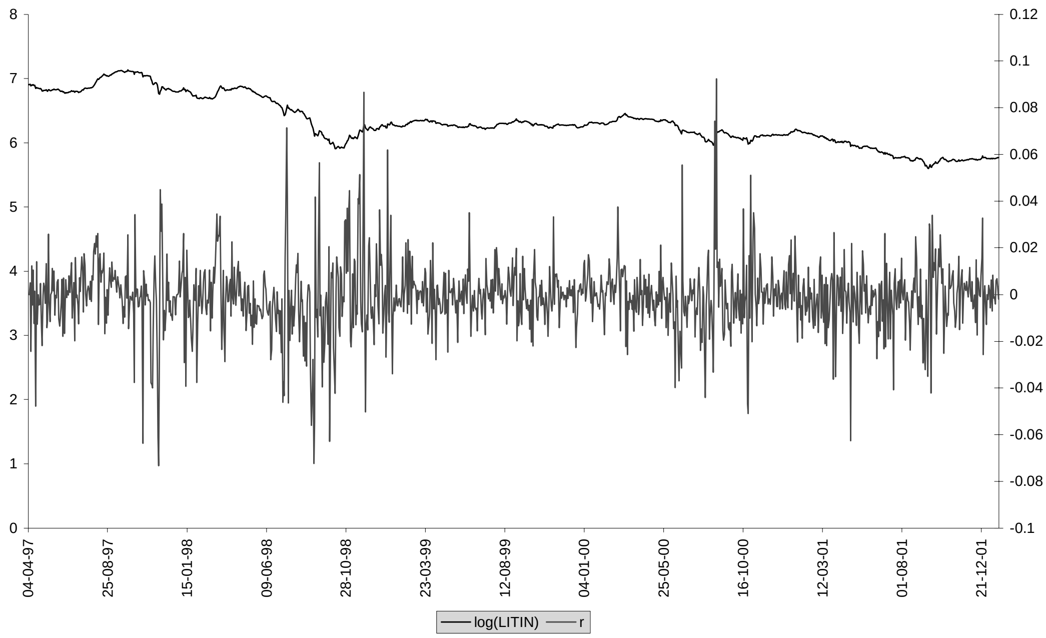
TALSE



DJRSE



LITIN



Estimated autocorrelations of index returns

Table 1, 2 and 3 report the auto-correlations and respective "static" (see Variance ratio test in the Annex) heteroscedasticity robust variance ratio statistics for returns of a daily Estonian Tallinn Stock Exchange index (TALSE), Latvian Riga Stock Exchange index (DJRSE[11]), and the Lithuanian Stock Exchange index (LITIN). The periods are selected on a yearly basis.

Table 1. Autocorrelation of daily index increments ($\varepsilon_t = log(TALSE_t) - log(TALSE_{t-1})$)

Price Data Period		Sample Size	Autocorrelations				Variance Ratios			
			ρ_1	ρ_3	ρ_7	ρ_{10}	M_1 (Prob.)	M_3 (Prob.)	M_7 (Prob.)	M_{10} (Prob.)
Total	06.03.96-01.02.02	1417	0.22	0.06	0.11	0.12	4.69	4.17	3.51	4.35
1.	06.03.96-01.02.97	150	0.40	-0.09*	-0.08*	0.17	2.35	1.48*	1.30*	1.54*
2.	01.03.97-01.02.98	253	0.19	0.05*	0.24	0.16	2.43	2.64	1.73*	2.79
3.	01.05.98-01.04.99	253	0.24	0.08*	0.02*	0.10*	4.27	2.86	1.92*	1.58*
4.	01.05.99-01.03.00	253	0.19	0.00*	0.04*	-0.03*	4.93	4.34	5.00	5.30
5.	01.00.10-01.02.01	254	0.04*	-0.07*	0.00*	-0.01*	0.77*	0.43*	0.42*	0.56*
6.	01.03.01-01.02.02	254	0.10*	0.10*	0.13	0.05*	2.19	3.18	4.78	5.46

* The null hypothesis on uncorrelated returns is accepted at the 5% significance level

[11] Dow Jones Riga Stock Exchange Index.

Table 2. Autocorrelation of daily index increments ($\varepsilon_t = log(DJRSE_t) - log(DJRSE_{t-1})$)

Price Data Period		Sample Size	Autocorrelations				Variance Ratios			
			ρ_1	ρ_3	ρ_7	ρ_{10}	M_1 (Prob.)	M_3 (Prob.)	M_7 (Prob.)	M_{10} (Prob.)
Total	04.02.96-01.02.02	1343	0.16	0.06	-0.01*	0.09	2.44	3.38	3.34	3.46
1.	04.02.96-01.02.97	88	0.00*	-0.03*	-0.12*	0.00*	0.00*	-0.77*	-0.75*	-0.86*
2.	01.03.97-01.05.98	247	0.20	0.11*	0.12	0.09*	1.72*	2.87	3.19	3.54
3.	01.06.98-01.04.99	249	0.23	0.01*	0.09*	-0.11*	2.72	2.24	3.30	3.77
4.	01.05.99-01.03.00	252	0.13	0.04*	-0.01*	0.10*	1.85*	2.54	3.06	3.56
5.	01.04.00-01.02.01	253	-0.07*	-0.10*	-0.03*	-0.04*	-0.45*	-0.10*	0.10*	0.18*
6.	01.03.01-01.02.02	254	0.20	0.09*	-0.13	0.21	3.03	5.58	3.08	1.98

* The null hypothesis on uncorrelated returns is accepted at the 5% significance level

Table 3. Autocorrelation of daily index increments ($\varepsilon_t = log(LITIN_t) - log(LITIN_{t-1})$)

Price Data Period		Sample Size	Autocorrelations				Variance Ratios			
			ρ_1	ρ_3	ρ_7	ρ_{10}	M_1 (Prob.)	M_3 (Prob.)	M_7 (Prob.)	M_{10} (Prob.)
Total	04.04.97-01.02.02	1206	0.22	-0.01*	0.07	0.02*	4.81	4.96	4.77	5.14
1.	04.04.97-01.05.98	192	0.28	-0.08*	0.20	0.06*	2.05	2.30	1.51*	2.29
2.	01.06.98-01.04.99	255	0.35	-0.03*	0.07*	-0.01*	4.61	4.64	4.72	4.75
3.	01.05.99-01.04.00	254	0.02*	0.07*	-0.08*	0.00*	-0.16*	-0.08*	-0.27*	0.38*
4.	01.05.00-01.02.01	252	0.21	-0.07*	0.02*	0.01*	4.00	4.53	3.91	3.97
5.	01.03.01-01.02.02	253	-0.02*	0.09*	0.05*	0.02*	-0.44*	-0.36*	1.05*	0.79*

* The null hypothesis on uncorrelated returns is accepted at the 5% significance level