Eesti Pank
Bank of Estonia

Forecasting Measures of Inflation for the Estonian Economy

Agostino Consolo

Working Paper Series
3/2006

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Abstract

The aim of this paper is to forecast some of the most important measures of inflation of the Estonian economy by making use of linear and non-linear models. Results from comparing classes of optimal models are similar to those in the forecasting literature. In particular, there are gains from using more sophisticated methods such as factor analysis and time-varying parameters methods.

Model discrimination is based on evaluation criteria which are computed by a real-time dynamic estimation procedure. Moreover, forecasts uncertainty is appropriately taken into account:*Fan Charts* can exhaustively describe the final output for what concerns out-of-sample forecasting.

JEL Code: C22, C32, C53, E31

Keywords: Estonian Economy, forecasting, inflation modelling

Author's e-mail address: email@agostino.it

*I am grateful to Marco Aiolfi, Andrea Carriero, Martin Lindpere and Massimiliano Marcellino for several conversations on some of the issues discussed in this paper. I acknowledge the financial support of Eesti Pank. Any remaining errors are my own responsibility.

Non-technical Summary

Nowadays, the vast literature on monetary policy has increased the knowledge of central bankers about the transmission mechanism and the understanding of how to stabilize the economy over business cycles. Recent evidence on the United States has shown that a possible explanation to the decrease in the volatility of the major macroeconomic indicators is due to a better monetary policy management. This is also confirmed by the work of Clarida, Galì and Gertler (2000) who provide compelling evidence that policy behavior matters in terms of macroeconomic stabilization. In the same vein, Svensson (1997) has shown how inflation forecast targeting can help policymakers to set the interest rate and to anchor private agents' expectations. The use of forecast-based rules was also found very powerful from new and growing literature on macroeconomics and learning (Evans and Honkapohja, 2000).

It becomes increasingly important for a central bank to create sophisticated tools to measure expectations both to retrieve private agents' expectations and to be able to forecast macroeconomic variables. This is done to build up a transparent mechanism for setting the policy rate. Moreover, the public awareness of a future path of the main macroeconomic variables can help both private agents and policymakers to quickly converge to the equilibrium value of the economy.

The paper aims at providing measures of expectations by estimating different econometric models given the current economic situation. These forecasts can also be part of regular publication by the monetary authority that wants to help private agents to form expectations about the state of the economy, or simply to let market operators know about central bank's evaluation of the state of the economy.

The model specification strategy borrows some important facts from the theory of business cycle. Referring to the original paper of Burns and Mitchell (1946), two key aspects need to be pointed out. According to them, macroeconomic time series at the business cycle frequency exhibit asymmetry and co-movements. As regards the asymmetry, the analysis of non-linear methods (NLM) enables to capture the dissimilarities during recessions and expansions phases. The co-movements among macro time series arise from the fact that only a few shocks are responsible for fluctuations of the economy as a whole. Therefore, all the series describing the entire system have their dynamics being generated by common shocks. Both of these crucial aspects are examined in the paper.

To somehow validate these theoretical facts, which support the idea of parameter instability, two important contributions to the analysis of time series are referred to. Recent works on US and euro area macroeconomic time series have shown the relevance of more sophisticated non-linear models in improving the forecasting performance. For example, Marcellino (2002) analyzes 500 European macroeconomic time series and shows that only for 30% of these series linear methods outperform the other methods. This fact can be justified by looking at the parameter instability which can be found in about 25% of macro variables for the euro area economy. A similar exercise is performed by Stock and Watson (1999) based on US macroeconomic time series. They have also found parameter instability; their results are in favor of NLMs and time-varying parameters (TVP) because of the gains in forecasting performance.

Contents

1. Introduction

The main concern of this paper is to model and forecast crucial macroeconomic variables by using state-of-the-art methodologies. The set of models employed will span from linear (LMs) to non-linear (NLMs) and time-varying parameters (TVPs) methods. The methodological approach is focused on model selection, which produces a rank of different models, on estimating the optimal model and on evaluating forecasts across all the optimal models.

The model selection is based on information criteria such as Akaike's, Schwarz's and Hannan-Quinn's: they are reliable tools to discriminate among models. In particular, they are very useful in forecasting analysis because they can avoid over-parameterization which improves in-sample fit performance but it goes against forecastability. They somehow reflect the parsimony principle in forecasting.

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To somehow validate these theoretical facts, which support the idea of parameter instability, two important contributions to the analysis of time series are referred to. Recent works on US and euro area macroeconomic time series have shown the relevance of more sophisticated non-linear models to improve forecasting performance. For example, Marcellino (2002) analyzes 500 European macroeconomic time series and he shows that only for 30% of these series LMs outperform the other methods. This fact can be justified by looking at parameters instability which can be found in about 25% of macro variables for the euro area economy. A similar exercise is worked out by Stock and Watson (1999a) based on US macroeconomic time series and they have also found parameters instability; their results are in favor of NLMs and TVPs because of the gains in forecasting performance.

It seems a natural consequence to make use of more sophisticated tools to improve the modelling strategy. Measuring non-constant and non-linear relationship can turn out to be very fruitful to better understand key macro variables. In this context, NLMs and TVPs methods have some comparative advantage with respect to LMs methods, but they suffer from short-sample estimation problems. What it is common in these setup is that NLMs and TVPs have better in-sample performance than LMs. This fact is mainly driven by their richer parameterization which increases the goodness of fit. Fortunately, information criteria can be employed to select among different models with different numbers of parameters by penalizing the increasing number of coefficients in the model. These procedures can avoid overfitting and permit to construct reliable measures of forecasting accuracy, however small sample bias of the estimates can crucially afflict non-linear methods and the real-time methodology applied in this work can further lead to poor forecasting results because it shortens the estimation sample. However, if combination of forecasts from different models is considered, an approach that is not studied in this paper, the construction of predictions from several non-nested models can improve the results from a single model. Four forecasting methods will be mainly worked out: linear and non-linear methods, time-varying coefficients methods and factors methods.

The class of linear methods groups univariate autoregressive models (AR), multivariate autoregressive models (VAR) and random walk models (RW). The last model is mainly used for forecasts comparison, even if results from comparing alternative methods to the RW model are not particularly useful because they depend on the variance of the underlying process for inflation as pointed out by Fisher et al. The NLMs class includes Logistic Smooth Transition (LSTR) and Logistic Smooth Transition Autoregressive (LSTAR). They differ from each other from the set of variables included in the specification of the Logistic function. In the class of time-varying parameters methods, purely autoregressive models and autoregressive models with exogenous variables will be specified and cast in a state-space representation. Given a state-space form, there are two possible estimation techniques: the first one assumes parameters (entries in the variance-covariance matrix for the errors in the transition and measurement equation) are known, while in the alternative one they are unknown and therefore they need to be estimated by maximizing the likelihood function. The latter will be implemented, even though is more demanding from a computational point of view. Furthermore, the process in the transition equation evolves according to the random walk hypothesis which turns out to be a standard specification used in the literature[1]. Factors methods

[1]See Hamilton (1994) and Kim & Nelson (1999) for similar applications of time-varying coefficients methods.

are used to derive leading indicators to include in the Phillips' curve specification. Originally, the Phillips' curve is defined as a linear relationship between the inflation rate and a distributed lags polynomial in inflation and unemployment (as deviations from an equilibrium value[2]). Recently, several theories have pointed out the use of other measures such as the output gap or the real marginal cost. The approach followed here is close to the analysis of Stock and Watson (1999c) where they find an index of economic activity to outperform (in a MSFE sense) any other leading indicators previously used. Factors are extracted from a data set which contains time series describing economic activity and used instead of unemployment or output gap.

Given the foregoing specifications, models and methods need to be compared. For each method, several optimal models[3] are retrieved depending on the set of exogenous variables added in the specification process. This variety of models derives from using different a-priori on the data generating process or from considering non-nested economic theories. This leads to do not concentrate on a particular model in each class, but on expanding the list of possible optimal models. The comparison exercise will not mainly be based on comparing models within the same class but all the models will be competing against each other.

The methodology applied for forecasts evaluation follows a real-time in-sample approach. In particular, real-time estimation and forecasts' construction leads to consider the correct information set available to the econometrician at the time s/he carries out the forecasting exercise. The metric used in this work to evaluate forecasts is based on recognizing the magnitude of the forecasting failure, that is, the larger the deviation from the actual value is, the greater the failure[4]. Metrics of this type are the *mean squared forecast error* (*MSFE*) and the *mean absolute forecast error* (*MAFE*). Models will be ranked by their *relative MSFE* and *MAFE* respectively, *RelMSFE* and *RelMAFE*, with respect to a benchmark one over different forecast horizons. For what concerns a statistical distribution of these relative measures of forecasts evaluation criteria, the usual asymptotic results cannot be applied in this context because of the short sample size and therefore common confidence sets retrieved by using the $\delta - method$ cannot be constructed. Bootstrap methods, that are not part of this paper, could be employed to create reliable confidence bands,

During the implementation of the codes, a VAR specification was tried, but convergence results were not achieved. An alternative setup where to introduce a VAR structure would be Factor Analysis which allows both loadings (data) and factors (coefficients) to be unknown.

[2]In the literature, the equilibrium value of unemployment, so-called NAIRU, was defined as being either constant or time-varying.

[3]Here, optimality derives from the use of lag-length selection criteria.

[4]An alternative evaluation method, that it is not implemented here, would be to consider a metric based on the direction of the forecast instead of the magnitude of the forecast error.

but they could drammatically increase computational time for non-linear and time-varying coefficients methods. On the other hand, a Bayesian approach in forecasting, Bayesian Model Averaging, would recommend to make use of all the models (across all the methods) and to construct a forecasts combination mechanism which weights forecasts from all the models analyzed. Stock and Watson (1999c) proceed in this fashion by adopting a Frequentist approach and they find that forecasting improvements are remarkable.

These measures of relative performance permit to discriminate and to rank among all the optimal models. As a final result, Phillips' curve, TVPs and VARs models[5] result to be among the best performing models and display *MSFE* (and *MAFE*) which significantly differ from the basic univariate autoregressive model. This is a result which points in favor of more complicated models in forecasting analysis. Moreover, out-of-sample forecasts are exhibited and forecasts' uncertainty is appropriately taken into account: tables with confidence sets and *Fan Charts* are displayed to improve the understanding of the forecasting exercise.

The remainder of the paper is organized as follows. In Section 2, the forecasting methodology is explained by considering the estimation procedure and methods for forecasts evaluation. A detailed description of methods' specification is discussed in Section 3. A brief outline of the data used in the forecasting exercise is presented in Section 4. The applied econometric exercise consisting of model selection, estimation and evaluation is fully described in Section 5. In this section results for some measures of inflation are shown and discussed. Section 6 concludes the work.

2. Forecasting Methodology

The general parametric formulation of the methods described in this paper can be summarized by this equation:

$$Y_t = f\left(W_t; \Theta_t\right) + \epsilon_t \tag{1}$$

where the function $f\left(\cdot\right)$ is known.

A forecasting model, given *eqn.1*, can be parsimoniously represented by the following equation:

[5]These results are based on a common sample starting from 1998. Smooth transition regression models (STRs) have also produced some good results but they need a longer sample size. For example, by running STRs and using a longer sample starting in 1995 improvements become remarkable.

$$\begin{aligned} Y_{t+h} &= f\left(W_t; \Theta_{h,t}\right) + \nu_{t+h} \\ E_t \nu_{t+h} &= 0 \end{aligned} \tag{2}$$

where W_t belongs to the information set of the econometrician and includes two main types of variables: X_t which is predetermined or weakly exogenous and Z_t which is, in general, strictly exogenous. X_t can be thought of being a collection of lagged values of Y_t while Z_t is defined as a leading indicator for the variable Y_t. Throughout the paper leading indicators which come from economic theory will be used and *ad-hoc* series will be drawn to capture a common pattern in the economy over the business cycle.

The forecasting equation, *eqn.3*, is quite general and delineate two dimensions of the analysis that will be used later on. Depending on how the function $f(\cdot)$ is specified different methods (e.g., linear and non-linear methods) can be defined and, given a functional form for $f(\cdot)$, numerous models within each method can show up depending on the number of lags, number of exogenous variables and degree of differentiation of the time series.

A note on the set of parameters, $\Theta_{h,t}$, is worth. *Eqn.3* make them very general allowing for time variation and forecast horizon dependency. Not all methods will have time-varying coefficients, instead the forecasting approach that will be worked out in this paper will deal with parameters that depend on the forecast horizon.

According to the forecasting equation, the *h-step* forecast reads:

$$Y_{t+h|t} = f\left(W_t; \Theta_{h,t}\right)$$

and the corresponding forecast error is:

$$\varepsilon_{t+h} = Y_{t+h} - Y_{t+h|t}$$

2.1. Forecasting by Dynamic Estimation

The forecasting approach pursued in this paper is based on a direct estimation of the h-step regression. What really matters in the forecasting procedure is how to calculate h-step forecasts for $h > 1$. The solution that is preferred in this context goes through a direct estimation of the multi-step relationship instead of substituting forward. Theoretically, this means choosing optimal coefficients by minimizing h-step ahead prediction error instead of 1-step ahead one (which comes out from the optimality principle).

Researchers who rely on dynamic estimation to forecast consider the possibility of misspecification errors induced by either unit roots (overdifferencing of a stationary time series) or omitted variables problem very relevant: *bias reduction*. Some other researchers agree on the optimality principle which delivers *inefficiency reduction*. What is better is an empirical question and crucially depends on the structure of the economy: it can happen that a subset of variables may affect a key macro variable in one regime, but it may be absent in another regime leading to an omitted variables problem.

Here, since sample data contain a brief history of the Estonian economy, misspecification errors become the main causes to consider the *bias reduction problem* more important than the gain in efficiency from minimizing the 1-step ahead forecast error.

2.2. Forecasts Evaluation

Forecasting models need to be evaluated according to some criteria. In this paper *MSFE* and *MAFE* will be used as metrics for forecasts accuracy. These criteria depend on the h-step ahead forecast error: how the forecast error is calculated crucially affects the evaluation process. A real-time procedure will be implemented to calculate the h-step ahead forecast error in this context.

Assume that the data sample size is T and split the sample such that estimation will be started by employing a subsample up to $FT < T$ only. A real time mechanism works as follows:

1. for $t = 1, \ldots, FT$, *eqn. 1* is estimated and the h-step ahead forecast, $Y_{t+h|FT}$, is retrieved:

$$\begin{aligned} Y_{t+h} &= f\left(W_t; \Theta_{h,t}^{FT}\right) + \nu_{t+h} \\ Y_{FT+h|FT} &= f\left(W_{FT}; \Theta_{h,FT}^{FT}\right) \\ \xi_{FT+h} &= Y_{FT+h} - Y_{FT+h|FT} \end{aligned}$$

2. by increasing FT of one step at the time up to $FT = (T - h)$ a series of $\left\{Y_{t+h|FT}\right\}_{t=FT}^{T-h}$ is calculated. Since forecasts were calculate in-sample, a series of h-step ahead forecast error, $\left\{\xi_{t+h}^{h}\right\}$, of length $(T - h - FT)$ can be derived as follows:

$$\xi_{t+h}^{h} = Y_{t+h} - Y_{t+h|FT}$$

for $t = FT, \ldots, T-h$.

3. Measures of forecasts accuracy can be derived by calculating *MSFE* and *MAFE* given the stochastic sequence $\left\{\xi_{t+h}^{h}\right\}$ and the following formulae:

$$MSFE_h = \frac{\xi_{t+h}'\xi_{t+h}}{(T-h-FT)} \tag{3}$$

$$MAFE_h = \sum\nolimits_{j=1}^{T-h-FT} \left| \frac{Y_{t+h}(j) - Y_{t+h|FT}(j)}{(T-h-FT)} \right| \tag{4}$$

4. Once the *MSFE* for each model, $MSFE_h^i$, is calculated and a benchmark models is defined (together with the respective *MSFE*, $MSFE_h^{bench}$), *Rel*$MSFE_h^i$ and *Rel*$MAFE_h^i$, *relative MSFE* and *MAFE* can be computed and they become a useful tool to compare forecasts among models from different methods or in the same class. For each model i the ratio is defined by:

$$\mathit{Rel}MSFE_h^i = \frac{MSFE_h^i}{MSFE_h^{bench}}$$

This ratio varies across forecast horizons implying that there might be models which outperform the benchmark model for some forecast horizons but not for some others.

2.3. Forecasting with Uncertainty

A forecasting methodology has to deal with the uncertainty in predicting future realizations; a measure of dispersion around the point forecast is thus necessary to enrich the appraisal of the forecast exercise. However, any measure of forecasts variability cannot give an exhaustive conjecture because it is not a probabilistic proposition about the event. A full description of forecasts that is in line with stochastic statements can be achieved by assuming a functional form of the distribution of forecast errors[6]. In this work a Gaussian functional form will be assumed for each of the h-step ahead forecast errors and the key parameters will be the h-step ahead forecasts and the $MSFE_h$ for the median and the variance respectively.

[6] An alternative way to introduce uncertainty is to assume a distribution for the conditional expectation of the regression model. In particular, we could also derive confidence sets by taking the distribution of the estimated coefficients.

Bootstrapping methods can also turn out to be useful in generating confidence sets.

Probabilistic statements can be managed by generating a Gaussian confidence set where the median is represented by the point forecast and several intervals, around the median, are constructed by assuming different probabilities of type-I error, α, and taking the percentile level attached to these probabilities.

For example, assume that $\omega_{t+h} \sim N\left(\mu^h, \sigma_\xi^h\right)$ is the random variable describing the process for the h-step ahead forecast where $\mu^h = Y_{t+h|t}$ and $\sigma_\xi^h = MSFE_h$ then confidence bands for different levels of α, $\left[\omega_{t+h}\left(\frac{\alpha}{2}\right),\right.$ $\left.\omega_{t+h}\left(1-\frac{\alpha}{2}\right)\right]$, can be determined as follows:

Table 1: Confidence Bands

Prob. of Type-I Error	*Lower Bound*	*Upper Bound*
$\alpha = 5\%$	$\Pr\left(\omega_{t+h} < LB_{\alpha=5}\right) = \frac{.05}{2}$	$\Pr\left(\omega_{t+h} < UB_{\alpha=5}\right) = 1 - \frac{.05}{2}$
$\alpha = 10\%$	$\Pr\left(\omega_{t+h} < LB_{\alpha=10}\right) = \frac{.1}{2}$	$\Pr\left(\omega_{t+h} < UB_{\alpha=10}\right) = 1 - \frac{.1}{2}$
$\alpha = 30\%$	$\Pr\left(\omega_{t+h} < LB_{\alpha=30}\right) = \frac{.3}{2}$	$\Pr\left(\omega_{t+h} < UB_{\alpha=30}\right) = 1 - \frac{.3}{2}$

The basic measure used to deal with uncertainty consists of the $MSFE_h$ which is retrieved by the real time procedure highlighted above. Since the $MSFE_h$ is not calculated by making use of a single observation of the h-step ahead forecast error, it can be seen as a robust metric of h-step ahead forecast uncertainty. Indeed, for every forecast horizon, the model validation phase consists of computing and storing a series of the h-step ahead forecast error together with the $MSFE_h$ while in the model forecasting phase $MSFE_h$ is employed for drawing confidence bands.

The *recursive* $MSFE_h$ represent an alternative evaluation criterion which reflects the forecasting accuracy of the model over time ($t = FT, ...,$ $T - h$).

$$\mathit{Rec}MSFE_{t,h} = \left(\xi_{t+h}^h\right)^2$$

While $MSFE_h$ is an average over forecast errors, $RecMSFE_{t,h}$ produces a view of the evolution of the magnitude of forecast failure period by period. According to the $RecMSFE_{t,h}$ metric, models could be discriminate if they had a relatively high $RecMSFE_{t,h}$ in the latest periods.

3. Methods Specification

3.1. Linear Methods

The class of linear models constitutes an important pillar in macroeconomics and forecasting. Even though linearity represents a simple assumption, the forecasting output from these methods turns out to outperform, along some dimensions, some other methods. Furthermore, these methods are easy to estimate and forecasting and computational time and efforts can be saved.

The main models described in this session comprehend the class of ARIMA models. The analysis will mainly concentrate on univariate and multivariate autoregressive models: AR and VAR.

The random walk or martingale hypothesis model (RW) sets the predicted future observations equal to the latest observation available in the information set: no-change forecast. The random walk model will be mainly recall for comparison purpose even though its forecasting performance can dramatically depend on the volatility of the underlying process as it is pointed out by Fisher et al (2002).

- The general formulation for the martingale hypothesis for the variable x_t reads:
$$E\left(x_{t+1} \mid \mathbb{I}_t\right) = x_t$$
where $\mathbb{I}_t$ is the information set of the econometrician.

- The univariate linear model, $AR\left(p\right)$, is defined as:
$$x_t = \rho\left(L\right) x_{t-1} + \varepsilon_t$$
and $\rho\left(L\right)$ is a distributed lags polynomial of order p. A slightly different specification is the one including exogenous variables, $z_t \in \mathbb{I}_t$:
$$x_t = \rho\left(L\right) x_{t-1} + \gamma\left(L\right) z_t + u_t$$
and it is briefly named $ARZ\left(p, q\right)$ where q is the order of the polynomial $\gamma\left(L\right)$.

- Let y_t be a stochastic vector of endogenous variables of size $(n, 1)$: the $VAR\left(p\right)$ representation reads:
$$y_t = A\left(L\right) y_{t-1} + \varepsilon_t$$

where $A(L)$ is a matrix-valued distributed lags polynomial of order p.

In this multivariate setup all entries are endogenous and describing a model economy coming from some basic economic theory. A particular attention has to be paid in defining the set of exogenous variables, $z_t \in \mathbb{I}_t$ (e.g., European inflation and production affecting prices and output of the Estonian economy is a causal relationship which can be easily assumed), otherwise problems related to endogeneity could arise.

The VAR specification with exogenous variables, $VARZ(p,q)$, reads:

$$y_t = A(L)\, y_{t-1} + B(L)\, z_t + u_t$$

and $B(L)$ is a matrix-valued distributed lags polynomial of order q.

3.2. Autoregressive Time-Varying Parameters

The previous specification for univariate autoregressive models assumes that $\rho(L)$ is time invariant. A more general setup can be made by allowing for the parameters to be time varying. What is behind such an assumption is the instable relationship underlying the economy. If this were true forecasts accuracy would increase due to a model specification which is more reliable.

The State-Space representation of the autoregressive time-varying parameters model can be represented as follows:

$$\begin{aligned} y_t &= \beta_t' x_t + \epsilon_t \\ \beta_t &= \beta_{t-1} + v_t \end{aligned}$$

where $\epsilon_t \sim N(0, R)$ and $v_t \sim N(0, Q)$. Here, time-varying coefficients evolve as a multivariate random walk process.

The variable x_t collects lags of the dependent variable, y_t, and the dynamic estimation procedure implies that x_t is further lagged by the forecast horizon. For some specifications, the vector x_t also includes some exogenous variables: all coefficients are thought of being time varying.

The estimation is performed by maximizing the log-likelihood of the State-Space representation with respect to the entries of the variance-covariance matrices R and Q. Since the process $\{\beta_t\}_{t=1}^{T}$ is not observable, the Kalman filter has to be employed to compute at each iteration the likelihood function.

An alternative estimation procedure would be to assume that the covariance matrices are known and to run the Kalman filter. Both ways are analyzed even though results from the maximization of the likelihood are the only ones to be shown.

3.3. Smooth Transition Regression

The main reason behind the use of non-linear models is that if the underlying process is not Gaussian, then in general the optimal forecast will not be linear. A leading example of non-linear models[7] is the threshold autoregressive model which reads:

$$y_t = \mathbb{I}\left(y_{t-1} > \bar{y}\right) \alpha\left(L\right) y_{t-1} + \mathbb{I}\left(y_{t-1} \leq \bar{y}\right) \beta\left(L\right) y_{t-1} + \varepsilon_t$$

where $\mathbb{I}\left(\cdot\right)$ is the indicator function which is equal to one whenever its argument is true and zero otherwise. The level $\bar{y}$ represents the threshold and it works as an unobservable variable which needs to be estimated. In this way the process y_t is assumed to have two regimes and consequentially $\alpha\left(L\right)$ and $\beta\left(L\right)$ capture two different dynamics. A more general setup would be to consider a smooth (continuous) function instead of the indicator function which is instead a step function and presents a discontinuity at $y_t = \bar{y}$.

Smooth transition methods belong to the class of non-linear parametric models and make use of a pre-specified function which models the transition. The idea supporting this methodology is based on the adjustment dynamic that involves parameters' change over time.

The Smooth Transition Regression, STR, reads:

$$y_t = \beta' x_t + \theta' x_t F\left(\gamma, z_t\right) + \varepsilon_t$$

where $F\left(\cdot\right)$ is the logistic function[8] which defines a smoothed transition over different regimes that are regulated by the process z_t. The $F\left(\cdot\right)$ mapping is specified as follows:

$$F\left(\gamma, z_t\right) = \frac{1}{1 + \exp\left(-\gamma' z_t\right)},$$

and following some recommendations by Granger-Terasvirta (1993) the standardized version of the previous equation is used:

$$F\left(\gamma, z_t\right) = \frac{1}{1 + \exp\left[-\gamma' \left(\frac{z_t - \mu_z}{\sigma_z}\right)\right]}$$

[7] A competitive model would be the Markov switching approach. In this setup, regimes are defined by an unobservable discrete variable which evolves according to a Markov probability matrix.

[8] The literature also describes the exponential function, $F\left(\gamma, z_t\right) = 1 - \exp\left(z_t' \left(diag\left(\gamma\right)\right) z_t\right)$, as a valid alternative for modelling the transition.

where μ_z and σ_z are respectively the mean and standard deviation of z_t. If z_t is a vector process, standardization is performed by taking each single variables and their own mean and variance.

3.4. Phillips' Curve

There is a vast literature on Phillips' curve both at the theoretical and empirical level. The general formulation is a linear relationship between inflation and unemployment; what such an equation predicts is a constant inflation rate whenever the unemployment rate is equal to a baseline unemployment level, the so-called NAIRU, which can also vary over time. Further developments have motivated, at least from a theoretical point of view[9], other measures of economic activity which could approximate reasonably well the short-run trade-off: output gap and real marginal costs are two prominent examples.

Stock and Watson (1999c) analyze a large set of leading indicators for inflation such as interest rates, monetary aggregates, stock prices, exchange rates and economic activity indicators. From their analysis, it comes out that the last group, indexes of economic activity, outperforms all the other measures from a forecasting point of view. In their paper, a lot of attention is paid to the analysis of parameters' instability as well. They convey that even though it exists, it turns out to be quantitatively negligible because impulse response functions from several regimes do not seem to statistically differ from each other.

Other empirical evaluations of the Phillips' curve tend to reject its usefulness, for example Atkeson and Ohanian (2001) prove forecasting overperfomance of a naive model with respect to the Phillips' curve. However, their results are not robust to changes in the estimation sample. In fact, Fisher et al (2002) find that overperfomance is due to the choice of the sample size together with the volatility of the underlying phenomenon which dramatically decays in the last twenty years. According to Fisher et al (2002), a metric which penalizes the wrong direction of forecasts instead of the magnitude of forecast errors recognizes the validity of the Phillips' curve in forecasting.

In this work, a two-step estimation procedure similar to the one that is implemented in Stock and Watson is run. Firstly, indicators of economic activity are retrieved by assuming that a set of variables describing the business cycle can be represented in a factor structure. Once factors, or indexes of economic activity, are available the Phillips' curve is estimated by specifying a linear regression model with an autoregressive component for the inflation measure,

[9] The New Keynesian Phillips Curve (NKPC) is an illustrative example where microfoundations derive from either *sticky prices adjustment* or *sticky information accumulation.*

x_t, and a distributed lags polynomial for the factors, F_t.

A general specification of the model to be estimated later on is:

$$\begin{aligned} Y_t &= \Lambda (L) F_t + \varepsilon_t \\ x_t &= \Phi (L) x_{t-1} + \Psi (L) F_t + v_t \end{aligned}$$

and (ε_t, v_s) are supposed to be uncorrelated at any lead and lag.

4. Data Source and Description

The set of time series used are extrapolated from different sources: Eesti Pank's data set mainly provides domestic measures of inflation series and harmonized series according to Eurostat's standards. The ECB data set collects key indicators which can be exploited to capture business cycle dynamic in the euro area. The IMF Commodity Price Index is seen as a measure of world price dynamic.

Time series included are observed at monthly frequency. The database collects series which differ in sample size: some of them start from January 1993, some others from January 1995 and only few of them run from January 1998.

For what concern seasonality, time series which are not seasonally adjusted have been smoothed by making use of a moving average filter with multiplicative factor. MA seasonal filtering assures constant multiplicative factors. This way of filtering is useful whenever forecasts need to be constructed back and a simple multiplication by these fixed factors can easily produce not-seasonally adjusted forecasts.

Furthermore, a small set of time series is not recorded at a monthly frequency. These series were transformed by assuming a linear growth over the quarter[10]: this transformation lets monthly and quarterly observations be equal to each other every quarter.

The main endogenous measures of price inflation are described in Table 2.

The set of remaining data can be broadly divided in three categories: domestic labor market and output indicators, foreign dynamic indicators and some other miscellaneous indexes which are fully described in the Appendix 1.

[10]Seasonal adjustments and frequency conversions were performed by using EViews®. A database is available where raw and adjusted series are available. Seasonal factors are also contained.

Table 2: Measures of Price Inflation

Measures :	*Brief Description :*	*Sample :*
EsCPI	Estonian Consumer Price Index	Jan93 - Jun04
EsCPI-NT	Estonian CPI (Non Tradable Goods)	Jan93 - Jun04
EsCPI-NTNR	Estonian CPI (Non Trad. & Non Reg.)	Jan93 - Jun04
EsHCPI	Estonia - Harmonized CPI (Eurostat)	Jan95 - Jun04
EsHCPI-CORE	Estonia - HCPI - Excl. Food and Energy	Jan98 - Jun04

5. Forecasting Inflation Measures

In this section of the paper, a basic description of the applied part of the project is produced. Given one of the measures of price inflation and the methods described in the previous paragraph, the analysis goes through *model selection* (optimal lag-length, constant, linear trend), *model validation* (calculation of the MSFE), *model estimation and forecasting* (recovering unknown parameters and constructing confidence set) and *model evaluation* (comparison among models).

5.1. Model Selection

The procedure of selecting a model in a class of models consists of detecting the optimal lag-length and whether intercept and/or linear trend has to be included in the estimation.

The evaluation criteria are based on information criteria which balance the increasing in the likelihood of the model, by augmenting the number of parameters, k, with a penalty function, Ξ. Depending on the information criterion (IC), penalty functions differ as follows:

- Akaike's IC : $\Xi_{AIC} = k\frac{2}{T}$
- Schwarz's IC : $\Xi_{BIC} = k\frac{\ln(T)}{T}$
- Hannan-Quinn's IC : $\Xi_{HQC} = k\frac{\ln(\ln(T))}{T}$

The mechanism implemented in this paper defines the optimal model given the pre-set information criterion (henceforth, AIC, BIC and HQC).

5.2. Model Specification

A description of the models used to forecast will be outlined. Within each method, there will be few optimal models which differ from each other because of variables considered in the specification. In particular, multivariate models were setup such that they can reflect some economic theory explaining causal relationship among the variables included. Given the sample size of the database, some possible specifications were not considered because of the limited amount of degrees of freedom for the estimation process. Clearly, the set of feasible specifications could be extended once more data will be available.

In what follows, forecasts for the series described in *Table 2* are produced. The generic variable "x_t" is used as a label to refer to the series in *Table 2* and it is intended to be the original variable with no transformation at all.

In the class of linear models, univariate and multivariate autoregressive models will be analyzed. These specifications will be enriched by making use of exogenous variables which can be dealt by employing multi-step estimation in the forecasting methodology. Time series that summarize the business cycle in the euro area will also be included to take inflationary pressures deriving from import prices or foreign demand into account.

5.2.1. AR Models

In the class of univariate linear models, autoregressive methods have good forecasting performance and together with the use of information criteria they can easily satisfy the parsimony principle so widespread in forecasting analysis. From a previous analysis of the series, all the measures of inflation reported in the table above are assumed to be unit root processes because of the results coming from unit roots tests[11]: results which turn out to be robust even if an alternative underlying data generating processes is considered.

The general formulation for the univariate autoregressive model that will be run for each series of price inflation reads:

$$\Delta \ln x_t = \kappa + \delta t + \phi (L) \Delta \ln x_t + \varepsilon_t$$

A similar but richer specification is the one where some exogenous variables are included. These exogenous variables can be thought of as leading indicators for inflation. Following Stock and Watson [1999], several subset of exogenous variables are considered and forecasts are worked out for these specifications as well:

[11] Unit roots tests were performed on EViews® and details can be found in the compendium Eviews' workfile.

$$\Delta \ln x_t = \kappa + \delta t + \varphi(L)\,\Delta \ln x_t + \gamma(L)\, z_t + u_t$$

Hence, given the assumption of non-stationarity which determines the degree of transformation of the original series and a selection criterion (*BIC*) the collection of **AR-Models** that will be estimated, forecast and evaluated is defined in Table 3:

Table 3: Collection of AR-Models that will be estimated, forecast and evaluated

Models	*Dependent*	*Exogenous Variables*
AR-01	$\Delta \ln x_t$	None
AR-02	$\Delta \ln x_t$	$\{\Delta \ln(\text{ComPrice})\ ;\ \text{EuULC}\ ;\ \text{EuIPI}\}$
AR-03	$\Delta \ln x_t$	$\{\Delta \ln(\text{ComPrice})\ ;\ \Delta \ln(\text{EuHICP})\ ;\ \text{EuIPI}\ ;\ \text{Spread}\}$
AR-04	$\Delta \ln x_t$	$\{\Delta \ln(\text{EsUne})\ ;\ \Delta \ln(\text{EsIPI})\ ;\ \Delta \ln(\text{EsWage})\}$

5.2.2. VAR Models

Multivariate linear models are specified as describing the economy as a whole. In choosing the set of variables to be included in a VAR model a little of economic theory is used to nest economic relationships.

The first set of endogenous variables is determined according to a*Micro view*: consumer price index, producer price index, real effective exchange rate and unit labor costs form the vector, Y_t^{Micro}, and they describe the interrelation in price formation.

The second group of endogenous variables reflects a *Macro view*. A representation depicted by industrial production, unemployment, consumer price and money, Y_t^{Macro}, refers to a macro model which includes a policy rule, an aggregate demand equation, a feasibility condition and an equation for the quantity theory with constant velocity. A VAR specification of this type is commonly used for policy analysis where impulse response functions from the estimated VAR are drawn to simulate the effects of policy shocks, technology shock, etc.

The starting system of equations for the vector autoregressive process is defined as:

$$Y_t = \mu + \zeta t + A(L)\, Y_{t-1} + \epsilon_t$$

Since the Estonian economy can be thought of being a small-open economy, an alternative specification would be one where some external factors are added to capture foreign dynamic effects. A distributed lag polynomial in the exogenous variable, Z_t, will group these facts.

$$Y_t = \mu + \zeta t + B(L) Y_{t-1} + \Psi(L) Z_t + v_t$$

Summarizing, groups of endogenous variables reflecting the two views are defined in Table 4.

Table 4: Groups of Endogenous Variables

Models	*Dependent Variables*
Macro View	$Y_t^{Macro} = \{\ln x_t \ ; \ \ln(\text{EsIPI}) \ ; \ln(\text{EsUne}) \ ; \ \ln(\text{M1})\}$
Micro View	$Y_t^{Micro} = \{\ln x_t \ ; \ \ln(\text{EsPPI}) \ ; \ln(\text{EsREER}) \ ; \ \ln(\text{EsULC})\}$

A full description of the VAR models to perform the forecasting exercise is defined in Table 5.

Table 5: Full Description of the VAR Models

Models	*Dependent*	*Exogenous Variables*
VAR-01	$\Delta \ln Y_t^{Micro}$	*None*
VAR-02	$\Delta \ln Y_t^{Micro}$	$\{\Delta \ln(\text{ComPrice}) \ ; \ \Delta \ln(\text{EuHICP}) \ ; \text{EuIPI} \ ; \text{Spread}\}$
VAR-03	$\Delta \ln Y_t^{Macro}$	*None*
VAR-04	$\Delta \ln Y_t^{Macro}$	$\{\Delta \ln(\text{ComPrice}) \ ; \ \Delta \ln(\text{EuHICP}) \ ; \text{EuIPI} \ ; \text{Spread}\}$

5.2.3. Logistic Smooth Transition

Now, non-linear methods are parsed by closely following the presentation in Granger and Terasvirta [1993]. The Logistic Smooth Transition Regression is the main method considered in this setup and reads:

$$\Delta \ln x_t = \kappa + \delta t + \beta(L) \Delta \ln x_{t-1} + F(z_t, \gamma) \theta(L) \Delta \ln x_{t-1} + \varepsilon_t$$

It is worth to notice the relevance of the variable z_t which crucially determines the transition dynamic together with the functional form of $F(\cdot)$[12]. To

[12]In considering the specification of the smooth transition regression, the logistic family is the only one to be taken into account; therefore the function $F(\cdot)$ is known.

capture such an important aspect, two alternative specifications are studied by changing the set of exogenous variables, z_t.

The first specification, **LSTAR**[13], uses lags of the dependent variables to construct z_t while the other one, **LSTR**, consists of using some valid leading indicators in the definition of z_t which could be worth in catching up the right turning points.

In sum, the two sets of exogenous variables are defined in Table 6.

Table 6: Exogenous Variables

Models	*Exogenous Variables*
LSTAR	$z_t = [x_{t-q}\ x_{t-q-1}\ x_{t-q-2}\ldots] \quad q = 1,\ldots,\bar{q}$
LSTR	$z_t = \{\Delta\ln(\text{ComPrice})\ ;\ \Delta\ln(\text{EuHICP})\ ;\ \text{EuIPI}\ ;\ \text{EuSpread}\}$

A log-levels specification is also run to check the robustness of the previous one. Obviously, non-linearity components can alter the usual analysis concerning unit roots tests. The model which uses levels of the process is specified as follows:

$$\ln x_t = \kappa + \delta t + \beta(L)\ln x_{t-1} + F(z_t,\gamma)\,\theta(L)\ln x_{t-1} + \varepsilon_t$$

and by collecting all the smooth transition regressions:

Table 7: Model by Collecting Smooth Transition Regressions

Models	*Dependent*	*Exogenous Variables*
LSTAR-01	$\Delta\ln x_t$	$z_t = \Delta\ln[x_{t-q}\ x_{t-q-1}\ x_{t-q-2}\ldots]$
LSTAR-02	$\ln x_t$	$z_t = \ln[x_{t-q}\ x_{t-q-1}\ x_{t-q-2}\ldots]$
LSTR-01	$\Delta\ln x_t$	$\{\Delta\ln(\text{ComPrice})\ ;\ \Delta\ln(\text{EuHICP})\ ;\ \text{EuIPI}\ ;\ \text{EuSpread}\}$
LSTR-02	$\ln x_t$	$\{\Delta\ln(\text{ComPrice})\ ;\ \Delta\ln(\text{EuHICP})\ ;\ \text{EuIPI}\ ;\ \text{EuSpread}\}$

5.2.4. Time-Varying Parameters

In this section time-varying parameters models for an univariate process will be analyzed. The starting point is to cast a general specification which also includes some exogenous variables into a state-space form. Coefficients, either

[13]Lags of the dependent variable have to be chosen appropriately because of the functional form which could lead to multi-collinearity of the regressors. By analyzing the estimation properties, it can be observed that the specification in levels is the most sensible to this choice.

for lagged values of the endogenous and for exogenous values are assumed to be time-varying.

The only parameters to be estimated are the entries in the var-covariance matrices of the measurement and transition error. The procedure sets up the likelihood function of the state-space and maximize it by numerical methods. The unobservable time-varying coefficients are retrieved by applying the Kalman filter.

The TVPs method reads:

$$\begin{aligned}
x_t &= \begin{bmatrix}\beta_t'(L) & \gamma_t'(L)\end{bmatrix}\begin{bmatrix}x_{t-1}\\ z_{t-1}\end{bmatrix} + \epsilon_t \\
\begin{bmatrix}\beta_{t+1}\\ \gamma_{t+1}\end{bmatrix} &= \begin{bmatrix}\beta_t\\ \gamma_t\end{bmatrix} + v_t \\
\epsilon_t &\sim N(0,R) \\
\nu_t &\sim N(0,Q) \\
& \quad \beta_0, V(\beta_0) \ given
\end{aligned}$$

The time-varying parameters specification assume a random walk evolution for the transition equation. A VAR structure like:

$$\Phi_t = F\Phi_{t-1} + u_t$$

was also implemented at the beginning. Unfortunately, such a state equation has induced instability in the all system leading to non convergence of the algorithm which was maximizing the log-likelihood function of the state-space model. Probably, in case of a small state-space representation a VAR mechanism might produce good results, but the procedure implemented here didn't permit that because of the automatic lag-length selection routine which seeks for the optimal model.

Depending on the set of exogenous variables considered, in Table 8 all the specification used in the forecasting exercise are collected:

5.2.5. Phillips' Curve: A Factor Analysis

A study by Stock and Watson showed that among all the possible leading indicators which can be used to forecast inflation, there is an index of economic activity which overperforms in terms of MSFE. There is a vast literature on forecasting inflation by using leading indicators which come from

Table 8: Specification used in the Forecasting

Models	*Dependent*	*Exogenous Variables*
AR-TVP-01	$\Delta \ln x_t$	*None*
ARZ-TVP-02	$\Delta \ln x_t$	$\{\Delta \ln (\text{ComPrice}) \ ; \ \Delta \ln (\text{EuHICP}) \ ; \text{EuSpread}\}$
ARZ-TVP-03	$\Delta \ln x_t$	$\{\Delta \ln (\text{EsPPI}) \ ; \ \Delta \ln (\text{EsULC}) \ ; \ \Delta \ln (\text{EuULC})\}$

the economic theory, but their work compares all these series and shows that the best indicators are the ones which are closely related to business cycle fluctuations[14].

In the analysis of Phillips' curve-based models these results are taken seriously and indexes of economic activity are constructed by factor analysis and its asymptotic counterpart, principal components analysis.

Given the usual definition of x_t as a measure of price inflation, the Phillips' curve is defined as follows:

$$\begin{aligned} X_t &= \Lambda (L) F_t + \varepsilon_t \\ \Delta \ln x_t &= \Phi (L) \Delta \ln x_{t-1} + \Psi (L) F_t + v_t \end{aligned}$$

where X_t, $(m, 1)$, is the set of time series from where factors F_t, $(k, 1)$ and $k < m$, are extracted. A full description of this data set can be found in the appendix; here it is worth to recall that they were chosen to reflect the state of Estonian economic activity over the business cycle[15], i.e. industrial production, credit flows, unemployment, money etc.

Table 9: Specification of the Model

Models	*Dependent*	*Exogenous Variables*
PHCu-01	$\Delta \ln x_t$	$F_t : k = 1$
PHCu-02	$\Delta \ln x_t$	$F_t : k = 3$
PHCu-03	$\Delta \ln x_t$	$\{F_t, F_{t-1}\} : k = 3$

[14]Other indicators include interest rates, monetary aggregates, exchange rates, etc..

[15]The selection procedure in this setup is constrained by data availability. We only use data from January 1998 to December 2003.

Factors introduce some peculiarities in the real time procedure implemented here which need to be taken into account to let the $MSFE_h$ unaffected from calculations. They thus have to be computed at each iteration to make sure that future information will not be included.

In the data set to calculate factors, time series for monetary aggregates were included. The only reason was to focus on money demand shocks which could be useful in interpreting business cycle fluctuations. However, they could also include money supply shocks but this might turn out to be a less desirable characteristic. As a robustness check, factors from a dataset excluding any measure of monetary aggregate were computed and compared with the previous ones. Since factors seem do not change across these two specifications, the former and richer data set is used in the forecasting exercise.

5.3. Forecasts Evaluation

Throughout this paper, a lot of attention was posed to evaluation criteria for forecasting models. This part of the work concentrates on in-sample comparison[16] while the next one will show results for out-of-sample forecasts evaluation

By running real-time in-sample dynamic estimation it is possible to construct forecasts for all the models considered in the previous session. Given the actual and predicted values at time $(t+h)\,, t = FT, \ldots, T-h$, the h-step ahead forecast errors can be computed. Once forecast errors are derived, metrics based on the h-step ahead forecast errors variability are calculated for each model. In particular, *MSFE* and *MAFE* are the two measures used for evaluation purpose. Firstly, a benchmark model is defined and secondly the remaining models are compared with respect to the benchmark one in terms of *MSFE* or *MAFE*. The benchmark model employed in this exercise, AR-01, somehow represents an optimal model from a forecasting point of view because it was defined according to BIC selection criterion[17].

In what follows, *Tables 10, 11, 12, 13* represent *RelMSFE* and *RelMAFE* for two main variables such as Estonian CPI and HCPI [18]. These tables are constructed by taking all the models and forecast horizons into account and therefore, for each variable, models' performance depends on the forecast horizon

[16]It has to be notice that a real-time procedure is employed for computing forecasts and therefore the term in-sample is not appropriate. This just wants to recall that actual data are available, letting comparison be feasible.

[17]An alternative that is less demanding could instead define as a benchmark model an autoregressive model without choosing the optimal lag length.

[18]To what concerns tables for the remaining variables, EsCPIT, EsCPINTNR and EsHCPI-Core, they can be found in the annex.

Table 10: (**EsCPI**): Relative MSFE (benchmark model is AR-01)

$Models \backslash FcstHorizon$	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6
AR-01 :	1.00	1.00	1.00	1.00	1.00	1.00
AR-02 :	1.06	1.30	1.52	1.46	1.34	1.30
AR-03 :	0.86	1.27	1.17	1.09	0.96	1.02
AR-04 :	0.84	1.12	1.04	1.08	1.23	1.28
VAR-01 :	0.62	0.92	0.69	0.79	0.56	0.67
VAR-02 :	0.61	0.86	0.68	0.85	0.77	0.95
VAR-03 :	0.81	1.11	1.03	1.09	1.19	1.28
VAR-04 :	0.88	1.13	1.06	1.15	1.23	1.30
LSTAR-01 :	0.84	0.91	0.89	1.03	1.15	1.22
LSTAR-02 :	1.95	5.66	6.86	9.30	12.53	19.93
LSTR-01 :	1.06	1.24	1.22	1.24	1.24	1.44
LSTR-02 :	2.04	6.69	10.82	17.93	25.87	36.98
TVP-01 :	0.84	1.33	1.08	1.21	1.18	1.22
TVP-02 :	0.70	1.17	1.06	1.35	1.45	1.22
TVP-03 :	0.66	0.98	0.70	0.93	0.78	1.16
PHCu-01 :	0.55	0.75	0.70	0.80	0.91	0.86
PHCu-02 :	0.52	0.76	0.78	0.82	0.96	0.92
PHCu-03 :	0.51	0.78	0.70	0.72	1.02	0.89

and on the metric used.

Tables 10 and 11 summarize *RelMSFE* and *RelMAFE*, respectively, for Estonian CPI inflation. It can be observed that the pattern between the two metrics is similar. By considering shorter forecast horizons, *ARZ, VARs, LSTAR, LSTR, TVPs and PHCu* models[19] give significative improvements over the benchmark model and gains can reach 50% in *RelMSFE* term. If instead the longest forecast horizon is considered, $h = 6$, some models fail to survive, but there are still some methods which have comparative advantage: Phillips' curve models and vector autoregression deliver very nice forecasting performance. The possible explanation of these results is likely to be driven by the common factors dynamic presents among the variables describing the economy. In fact, both PHCu and VARs can deal with common driving forces. For what concern VAR analysis, threre could be misspecification errors due to the badly specified error-correction representation. By allowing for dynamic estimation techniques, problems related to misspecification can be partially managed.

It is worth to notice that these comparisons span on a limited sample size. By enlarging the sample period, for example starting from 1995 instead of 1998, non-linear models such as STR improve considerably: in what follows a common sample was employed because of evaluation purposes.

[19]For a clear interpretation of these models, the section on model specification fully describes the endogenous and exogenous variables employed in the computation.

Table 11: (**EsCPI**): Relative MSFE (benchmark model is AR-01)

$Models \backslash FcstHorizon$	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6
AR-01 :	1.00	1.00	1.00	1.00	1.00	1.00
AR-02 :	1.05	1.15	1.24	1.20	1.19	1.21
AR-03 :	0.95	1.15	1.05	1.02	0.97	1.02
AR-04 :	0.85	0.98	0.99	0.92	1.01	1.10
VAR-01 :	0.81	1.05	0.84	0.87	0.76	0.84
VAR-02 :	0.79	0.98	0.81	0.87	0.76	0.92
VAR-03 :	0.82	0.96	0.99	0.92	0.97	1.08
VAR-04 :	0.89	1.00	1.01	1.03	1.08	1.11
LSTAR-01 :	0.91	0.96	0.90	0.93	0.99	1.03
LSTAR-02 :	1.22	2.60	2.83	3.05	3.60	4.80
LSTR-01 :	0.98	1.11	1.14	1.03	1.03	1.20
LSTR-02 :	1.26	2.07	2.54	2.35	2.64	3.35
TVP-01 :	0.88	1.19	1.00	0.99	1.02	1.02
TVP-02 :	0.82	1.12	1.04	1.10	1.21	1.12
TVP-03 :	0.83	1.07	0.89	0.97	0.87	1.05
PHCu-01 :	0.68	0.83	0.84	0.77	0.85	0.87
PHCu-02 :	0.66	0.81	0.88	0.77	0.89	0.87
PHCu-03 :	0.67	0.87	0.83	0.78	0.89	0.88

Tables 12 and 13 depict an analogous situation for the Estonian HCPI inflation. There are minor differences such as the increasing performance of TVPs models in the longest horizon and some deterioration concerning smooth transition models. Comparing PHCu and VARs, the former seems to perform a little bit better for the Estonian HCPI than CPI inflation. Such a description can be extended to all the other variables in *Table 2*: for convenience, tables for the other measures of inflation are put in the appendix.

Hence, from the forecasts evaluation part it appears that there are rooms to improve over the simple univariate model by making use of multivariate linear methods and more sophisticated ones such as TVPs, STRs[20] and PHCu methods.

5.4. Out-of-Sample Forecasting

To conclude the exposition of the forecasting exercise, tables and charts for some variables and models are shown which describe out-of-sample forecasts. Since it wasn't possible to include all the pictures and tables derived from the computational procedures, here some examples are shown.

[20]This class of models suffers from short sample size. In fact, some of these models can be worked out with a data set starting from 1995. In this case, *RelMSFE* favors LSTAR models as well.

Table 12: **(EsHCPI)** : Relative MSFE (benchmark model is AR-01)

$(Models \backslash FcstHorizon$	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6
AR-01 :	1.00	1.00	1.00	1.00	1.00	1.00
AR-02 :	1.03	1.14	1.10	1.11	1.10	1.00
AR-03 :	1.03	1.04	0.97	0.94	1.04	1.06
AR-04 :	0.97	1.06	1.05	1.00	0.97	1.08
VAR-01 :	0.52	0.59	0.52	0.31	0.23	0.39
VAR-02 :	0.44	0.49	0.40	0.44	0.47	0.36
VAR-03 :	0.99	1.10	1.03	1.02	0.98	1.07
VAR-04 :	0.99	1.01	0.94	0.99	1.14	1.06
LSTAR-01 :	1.03	0.99	1.06	1.03	1.06	1.01
LSTAR-02 :	2.78	4.55	4.09	4.52	5.68	7.26
LSTR-01 :	0.93	1.00	1.01	1.11	1.03	1.08
LSTR-02 :	2.78	4.77	5.63	7.10	8.32	16.43
TVP-01 :	1.19	1.26	1.19	1.05	1.32	1.37
TVP-02 :	0.95	1.37	1.58	1.72	1.44	1.32
TVP-03 :	0.45	0.45	0.40	0.36	0.45	0.53
PHCu-01 :	0.42	0.42	0.43	0.41	0.45	0.48
PHCu-02 :	0.40	0.45	0.52	0.42	0.50	0.52
PHCu-03 :	0.40	0.49	0.42	0.42	0.56	0.47

Table 13: **(EsHCPI)** : Relative MAFE (benchmark model is AR-01)

$Models \backslash FcstHorizon$	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6
AR-01 :	1.00	1.00	1.00	1.00	1.00	1.00
AR-02 :	1.02	1.04	1.05	1.06	1.08	1.10
AR-03 :	1.18	1.10	1.02	1.04	1.09	1.09
AR-04 :	0.98	1.01	1.02	0.94	1.02	0.96
VAR-01 :	0.89	1.00	0.93	0.75	0.67	0.85
VAR-02 :	0.82	0.92	0.83	0.85	0.89	0.73
VAR-03 :	0.96	0.99	1.02	0.95	1.02	1.01
VAR-04 :	1.16	1.04	1.01	1.07	1.23	1.13
LSTAR-01 :	0.99	1.01	1.05	1.02	1.04	1.00
LSTAR-02 :	1.58	2.31	2.48	2.66	3.08	3.65
LSTR-01 :	0.92	1.04	1.07	1.11	1.04	1.10
LSTR-02 :	1.62	2.44	2.92	3.42	3.75	4.92
TVP-01 :	1.08	1.13	1.16	1.02	1.27	1.30
TVP-02 :	0.98	1.42	1.45	1.61	1.40	1.40
TVP-03 :	0.88	0.88	0.82	0.80	0.90	0.95
PHCu-01 :	0.79	0.79	0.78	0.71	0.73	0.84
PHCu-02 :	0.77	0.76	0.79	0.74	0.82	0.91
PHCu-03 :	0.71	0.76	0.77	0.76	0.94	0.71

Tables summarize point forecasts together with lower and upper bounds. These bounds are constructed by taking different levels of uncertainty into account. In the previous session it was depicted how to generate such bounds. In particular, $Pr\,(Type\ I) = 20$ means that the h-step ahead forecast will be in that interval with a level of confidence of 80% [21]. By increasing the level of type-I error, confidence sets get shorter and shorter.

Table 14: Estonian CPI Inflation; Model: VAR01

$Confidence\ Set \backslash FcstHorizon$	**h = 1**	**h = 2**	**h = 3**
Median Forecast	0.88	1.50	2.01
$Pr\,(Type\ I) = 20$	(0.34 , 1.50)	(0.33 , 2.72)	(0.23 , 3.82)
$Pr\,(Type\ I) = 30$	(0.44 , 1.38)	(0.54 , 2.50)	(0.54 , 3.49)
$Pr\,(Type\ I) = 40$	(0.52 , 1.31)	(0.72 , 2.33)	(0.82 , 3.22)
$Pr\,(Type\ I) = 60$	(0.67 , 1.12)	(1.03 , 1.98)	(1.30 , 2.72)
$Pr\,(Type\ I) = 80$	(0.77 , 0.99)	(1.26 , 1.74)	(1.64 , 2.35)
$Pr\,(Type\ I) = 90$	(0.83 , 0.93)	(1.39 , 1.62)	(1.82 , 2.18)
$Confidence\ Set \backslash FcstHorizon$	**h = 4**	**h = 5**	**h = 6**
Median Forecast	1.89	2.10	1.86
$Pr\,(Type\ I) = 20$	(-0.46 , 4.35)	(-0.73 , 5.12)	(-1.45 , 5.45)
$Pr\,(Type\ I) = 30$	(-0.02 , 3.91)	(-0.19 , 4.58)	(-0.83 , 4.81)
$Pr\,(Type\ I) = 40$	(0.33 , 3.55)	(0.24 , 4.12)	(-0.33 , 4.25)
$Pr\,(Type\ I) = 60$	(0.97 , 2.89)	(0.99 , 3.32)	(0.54 , 3.31)
$Pr\,(Type\ I) = 80$	(1.43 , 2.39)	(1.55 , 2.72)	(1.21 , 2.59)
$Pr\,(Type\ I) = 90$	(1.67 , 2.15)	(1.86 , 2.42)	(1.58 , 2.24)

Last observation : March 2004

Figure 1, Figure 2, Figure 3 and Figure 4, better known as *Fan Charts*, are the visual counterpart of these tables. Fan Charts report few historical observations plus h-step ahead forecasts with confidence bands for different probabilities of type-I error. In this case, they are generated by making use of a symmetric distribution, but it would also be more realistic to draw them from a skewed distribution which reflects that some events have a larger probability mass than others: think of a supply shock (for example, a cost-push shock) implying that it is more likely an increase in price inflation.

The representation of the forecasting output with these tools permit an easier interpretation in terms of uncertainty and it also constitutes a more transparent way to communicate inflation expectations.

[21] This is rather a bayesian interpretation of a confidence set instead of a frequentist one.

Table 15: Estonian HCPI Inflation; Model: VAR01

$Confidence\ Set \backslash FcstHorizon$	**h = 1**	**h = 2**	**h = 3**
Median Forecast	1.06	1.09	1.22
$Pr\,(Type\ I) = 20$	(0.73 , 1.42)	(0.36 , 1.86)	(0.11 , 2.36)
$Pr\,(Type\ I) = 30$	(0.79 , 1.36)	(0.48 , 1.72)	(0.30 , 2.15)
$Pr\,(Type\ I) = 40$	(0.84 , 1.31)	(0.60 , 1.61)	(0.48 , 1.99)
$Pr\,(Type\ I) = 60$	(0.92 , 1.22)	(0.79 , 1.43)	(0.75 , 1.70)
$Pr\,(Type\ I) = 80$	(1.00 , 1.15)	(0.96 , 1.25)	(1.00 , 1.44)
$Pr\,(Type\ I) = 90$	(1.04 , 1.11)	(1.04 , 1.17)	(1.12 , 1.33)
$Confidence\ Set \backslash FcstHorizon$	**h = 4**	**h = 5**	**h = 6**
Median Forecast	0.95	0.63	0.45
$Pr\,(Type\ I) = 20$	(-0.43 , 2.35)	(-1.00 , 2.27)	(-1.50 , 2.43)
$Pr\,(Type\ I) = 30$	(-0.20 , 2.10)	(-0.72 , 1.97)	(-1.16 , 2.06)
$Pr\,(Type\ I) = 40$	(0.03 , 1.90)	(-0.45 , 1.72)	(-0.84 , 1.77)
$Pr\,(Type\ I) = 60$	(0.38 , 1.54)	(-0.05 , 1.30)	(-0.37 , 1.26)
$Pr\,(Type\ I) = 80$	(0.68 , 1.22)	(0.30 , 0.95)	(0.05 , 0.82)
$Pr\,(Type\ I) = 90$	(0.82 , 1.08)	(0.46 , 0.78)	(0.25 , 0.63)

Last observation : March 2004

Table 16: Estonian HCPI Inflation; Model: AR03

Estonian HCPI Inflation | *Model: AR03*

$Confidence\ Set \backslash FcstHorizon$	**h = 1**	**h = 2**	**h = 3**
Median Forecast	5.42	5.55	5.47
$Pr\,(Type\ I) = 20$	(4.89 , 5.92)	(4.46 , 6.65)	(3.85 , 7.12)
$Pr\,(Type\ I) = 30$	(4.98 , 5.80)	(4.68 , 6.41)	(4.18 , 6.78)
$Pr\,(Type\ I) = 40$	(5.05 , 5.73)	(4.84 , 6.25)	(4.43 , 6.53)
$Pr\,(Type\ I) = 60$	(5.19 , 5.61)	(5.10 , 6.00)	(4.81 , 6.14)
$Pr\,(Type\ I) = 80$	(5.31 , 5.51)	(5.36 , 5.77)	(5.19 , 5.80)
$Pr\,(Type\ I) = 90$	(5.36 , 5.45)	(5.46 , 5.66)	(5.35 , 5.63)
$Confidence\ Set \backslash FcstHorizon$	**h = 4**	**h = 5**	**h = 6**
Median Forecast	5.71	5.85	6.27
$Pr\,(Type\ I) = 20$	(3.64 , 7.85)	(3.25 , 8.53)	(3.10 , 9.54)
$Pr\,(Type\ I) = 30$	(4.05 , 7.43)	(3.78 , 8.03)	(3.72 , 8.92)
$Pr\,(Type\ I) = 40$	(4.38 , 7.09)	(4.18 , 7.61)	(4.22 , 8.40)
$Pr\,(Type\ I) = 60$	(4.89 , 6.60)	(4.81 , 6.99)	(4.97 , 7.63)
$Pr\,(Type\ I) = 80$	(5.36 , 6.15)	(5.38 , 6.40)	(5.67 , 6.93)
$Pr\,(Type\ I) = 90$	(5.56 , 5.93)	(5.63 , 6.11)	(5.99 , 6.59)

Last observation June 2004

Table 17: Estonian HCPI Inflation; Model: LSTAR01

$Confidence\ Set \backslash FcstHorizon$	**h = 1**	**h = 2**	**h = 3**
Median Forecast	4.86	5.02	5.07
$Pr\,(Type\ I) = 20$	(4.30 , 5.42)	(3.91 , 6.10)	(3.36 , 6.76)
$Pr\,(Type\ I) = 30$	(4.39 , 5.30)	(4.13 , 5.87)	(3.69 , 6.41)
$Pr\,(Type\ I) = 40$	(4.48 , 5.23)	(4.31 , 5.74)	(3.95 , 6.17)
$Pr\,(Type\ I) = 60$	(4.61 , 5.08)	(4.59 , 5.46)	(4.39 , 5.75)
$Pr\,(Type\ I) = 80$	(4.74 , 4.98)	(4.83 , 5.26)	(4.76 , 5.42)
$Pr\,(Type\ I) = 90$	(4.81 , 4.91)	(4.94 , 5.13)	(4.92 , 5.23)
$Confidence\ Set \backslash FcstHorizon$	**h = 4**	**h = 5**	**h = 6**
Median Forecast	5.24	5.17	5.36
$Pr\,(Type\ I) = 20$	(2.96 , 7.49)	(2.32 , 8.06)	(1.99 , 8.87)
$Pr\,(Type\ I) = 30$	(3.42 , 7.04)	(2.87 , 7.49)	(2.63 , 8.16)
$Pr\,(Type\ I) = 40$	(3.76 , 6.72)	(3.31 , 7.08)	(3.15 , 7.67)
$Pr\,(Type\ I) = 60$	(4.33 , 6.14)	(4.05 , 6.34)	(4.01 , 6.79)
$Pr\,(Type\ I) = 80$	(4.81 , 5.70)	(4.64 , 5.76)	(4.71 , 6.08)
$Pr\,(Type\ I) = 90$	(5.04 , 5.45)	(4.92 , 5.47)	(5.06 , 5.73)

Last observation : June 2004

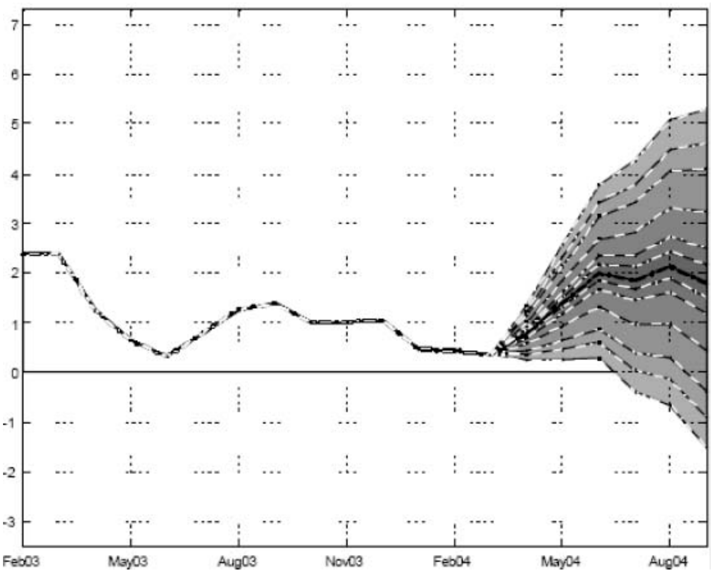

Figure 1: Series: Estonian CPI Inflation (YoY % change); Model: VAR-01

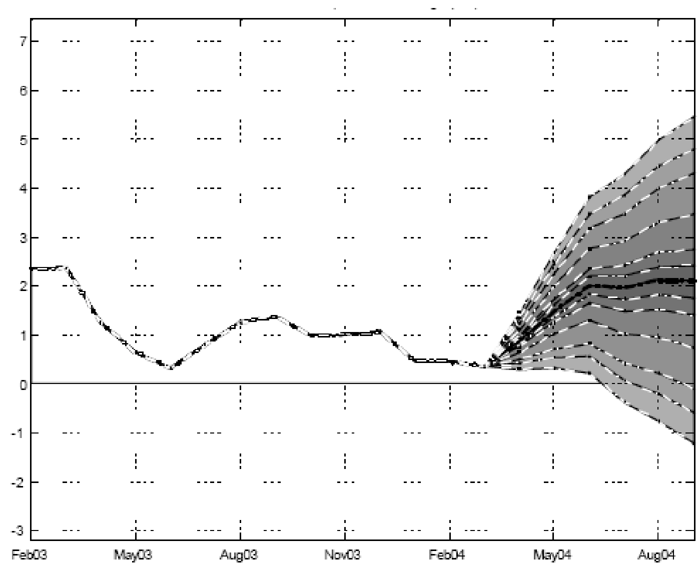

Figure 2: Series: Estonian CPI Inflation (YoY % change); Model: TVP-03

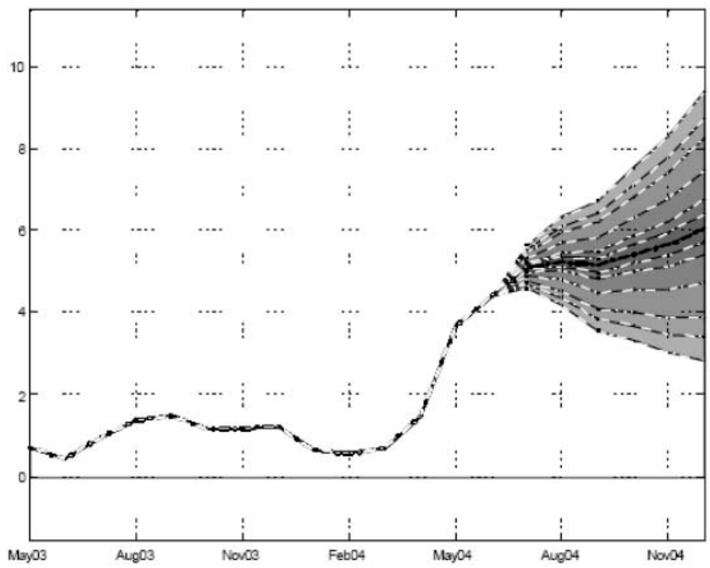

Figure 3: Series: Estonian HCPI Inflation (YoY % change); Model: AR-03

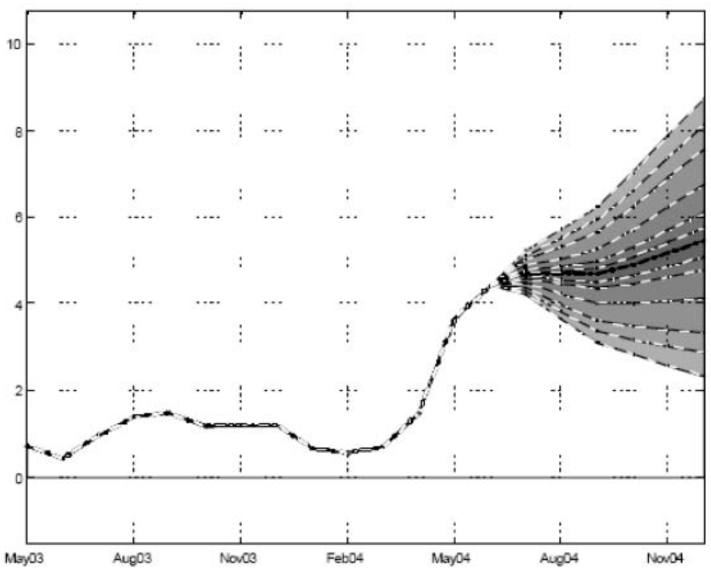

Figure 4: Series: Estonian HCPI Inflation (YoY % change); Model: LSTAR-01

6. Conclusions

The target of this paper is to model and forecast measures of inflation for the Estonian economy. The analysis goes through few steps which constitute the structure of the general procedure that is also implemented in the compendium Matlab codes. The basic underlying idea is to use different classes of models from which to retrieve an optimal model from each class that will be estimated and forecast (both in sample and out of sample).

In-sample forecasts are used to construct evaluation criteria to compare among all the optimal models. Out-of-sample forecasts are depicted by means of *Fan Charts* and *Table with Confidence Sets* which become an invaluable tools in forecasting analysis.

The main contribution of the paper is to show there are comparative performances across different methods that do not strictly favor just some of them. Therefore, models' over-performance is analyzed across two dimensions: the forecast horizon and the number of measures of inflation. The set of best performing models changes a little bit when the forecast horizon increases but it is almost the same when different measures of inflation are considered.

Since the set of best performing model change over some dimension, a caveat of this paper might be having not considered the study of forecast combination which could improve upon the output of these single models. As it is shown by Stock and Watson (1999c) for the US economy, there are forecasting gains in taking forecasts from different models into account and combining them by a weighting function. Even though there is no theoretical explanation in this paper why that could turn out to be fruitful, it can be easily implemented by recalling the Matlab codes and appropriately defining a weighting function.

Furthermore, it is worth to point out that more sophisticated methods, non-linear and time-varying parameters models, are penalized by a sample period which turns out to be rather short so that the estimation and forecast evaluation have been conducted on few observations. A longer sample size could improve their own performance and point even more in their favor with respect to other models. It needs to be mentioned that these results are also constrained by the limited amount of series employed and further developments can be done.

To what concerns Phillips' curve forecasts, the use of the factor analysis improves their performance and so accurate results reflect the widespread use among applied macroeconomic forecasters who employ the Phillips' curve for short-run inflation forecasting.

To conclude, the analysis of this paper presents a fruitful implementation of several methodologies in forecasting one of the most important variables of the business cycle.

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Appendix 1. Absolute MAFE and MSFE

EsCPI : Absolute MSFE

$Models$	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6
RW-00 :	0.0000413	0.0000424	0.0000431	0.0000441	0.0000439	0.0000441
AR-01 :	0.0000255	0.0000255	0.0000281	0.0000273	0.0000315	0.0000284
AR-02 :	0.0000252	0.0000269	0.0000290	0.0000282	0.0000311	0.0000293
AR-03 :	0.0000280	0.0000276	0.0000292	0.0000291	0.0000289	0.0000293
AR-04 :	0.0000267	0.0000275	0.0000297	0.0000296	0.0000331	0.0000310
VAR-01 :	0.0000184	0.0000208	0.0000212	0.0000186	0.0000174	0.0000197
VAR-02 :	0.0000197	0.0000177	0.0000189	0.0000204	0.0000221	0.0000256
VAR-03 :	0.0000257	0.0000271	0.0000296	0.0000299	0.0000320	0.0000309
VAR-04 :	0.0000280	0.0000278	0.0000304	0.0000315	0.0000332	0.0000316
LSTAR-01 :	0.0000255	0.0000280	0.0000297	0.0000274	0.0000360	0.0000307
LSTAR-02 :	0.0000650	0.0001219	0.0001546	0.0001410	0.0001471	0.0001554
LSTR-01 :	0.0000257	0.0000250	0.0000275	0.0000280	0.0000337	0.0000295
LSTR-02 :	0.0000674	0.0001286	0.0001682	0.0001728	0.0001860	0.0002324
TVP-01 :	0.0000303	0.0000310	0.0000360	0.0000304	0.0000318	0.0000248
TVP-02 :	0.0000230	0.0000268	0.0000328	0.0000293	0.0000304	0.0000307
TVP-03 :	0.0000199	0.0000203	0.0000211	0.0000170	0.0000180	0.0000131
PHCu-01 :	0.0000175	0.0000183	0.0000201	0.0000219	0.0000247	0.0000208
PHCu-02 :	0.0000166	0.0000186	0.0000223	0.0000224	0.0000260	0.0000224
PHCu-03 :	0.0000164	0.0000190	0.0000201	0.0000197	0.0000276	0.0000216

EsHCPI : Absolute MSFE

$Models$	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6
RW-00 :	0.0000241	0.0000248	0.0000254	0.0000259	0.0000267	0.0000273
AR-01 :	0.0000142	0.0000156	0.0000159	0.0000150	0.0000168	0.0000163
AR-02 :	0.0000146	0.0000178	0.0000175	0.0000166	0.0000185	0.0000163
AR-03 :	0.0000146	0.0000163	0.0000154	0.0000140	0.0000174	0.0000172
AR-04 :	0.0000138	0.0000166	0.0000166	0.0000150	0.0000162	0.0000175
VAR-01 :	0.0000073	0.0000092	0.0000082	0.0000046	0.0000038	0.0000064
VAR-02 :	0.0000063	0.0000076	0.0000063	0.0000067	0.0000079	0.0000058
VAR-03 :	0.0000140	0.0000173	0.0000164	0.0000153	0.0000164	0.0000174
VAR-04 :	0.0000141	0.0000159	0.0000149	0.0000148	0.0000192	0.0000172
LSTAR-01 :	0.0000153	0.0000182	0.0000191	0.0000211	0.0000247	0.0000218
LSTAR-02 :	0.0000417	0.0000664	0.0000889	0.0000892	0.0000967	0.0001330
LSTR-01 :	0.0000133	0.0000156	0.0000158	0.0000166	0.0000172	0.0000177
LSTR-02 :	0.0000380	0.0000743	0.0000894	0.0001061	0.0001399	0.0002679
TVP-01 :	0.0000169	0.0000197	0.0000189	0.0000157	0.0000223	0.0000223
TVP-02 :	0.0000135	0.0000214	0.0000251	0.0000257	0.0000242	0.0000214
TVP-03 :	0.0000064	0.0000071	0.0000064	0.0000054	0.0000075	0.0000086
PHCu-01 :	0.0000059	0.0000065	0.0000069	0.0000061	0.0000076	0.0000078
PHCu-02 :	0.0000057	0.0000070	0.0000082	0.0000062	0.0000085	0.0000085
PHCu-03 :	0.0000057	0.0000077	0.0000067	0.0000063	0.0000095	0.0000076

EsCPI : Absolute MAFE

$Models$	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6
RW-00 :	0.0034676	0.0034273	0.0035192	0.0035826	0.0036206	0.0036861
AR-01 :	0.0035831	0.0035161	0.0038166	0.0036464	0.0039011	0.0038234
AR-02 :	0.0035670	0.0036162	0.0039888	0.0039460	0.0042651	0.0039547
AR-03 :	0.0039249	0.0037878	0.0039397	0.0041932	0.0038870	0.0040538
AR-04 :	0.0037730	0.0036115	0.0039286	0.0038537	0.0039951	0.0040714
VAR-01 :	0.0033594	0.0036630	0.0034865	0.0032869	0.0032857	0.0034861
VAR-02 :	0.0033183	0.0032471	0.0031790	0.0034492	0.0036131	0.0038591
VAR-03 :	0.0036557	0.0035083	0.0039267	0.0038517	0.0038457	0.0039769
VAR-04 :	0.0039437	0.0036639	0.0040014	0.0043265	0.0042568	0.0041213
LSTAR-01 :	0.0035076	0.0037216	0.0040063	0.0035879	0.0044269	0.0042649
LSTAR-02 :	0.0061270	0.0089607	0.0098282	0.0091488	0.0102261	0.0104811
LSTR-01 :	0.0036775	0.0035007	0.0037808	0.0037517	0.0042802	0.0039430
LSTR-02 :	0.0062370	0.0092088	0.0101213	0.0102661	0.0113439	0.0121357
TVP-01 :	0.0039799	0.0039028	0.0044389	0.0040666	0.0036206	0.0033884
TVP-02 :	0.0034919	0.0037152	0.0041061	0.0039383	0.0040137	0.0040663
TVP-03 :	0.0035477	0.0035588	0.0036068	0.0032920	0.0033144	0.0028199
PHCu-01 :	0.0030291	0.0030357	0.0033318	0.0032110	0.0033461	0.0032079
PHCu-02 :	0.0029413	0.0029847	0.0034958	0.0032218	0.0035111	0.0032332
PHCu-03 :	0.0029705	0.0031866	0.0032867	0.0032649	0.0035159	0.0032435

EsHCPI : Absolute MAFE

$Models$	h = 1	h = 2	h = 3	h = 4	h = 5	h = 6
RW-00 :	0.0024159	0.0023654	0.0023590	0.0024092	0.0023951	0.0024305
AR-01 :	0.0024687	0.0024121	0.0023840	0.0022804	0.0024342	0.0023562
AR-02 :	0.0025128	0.0025073	0.0025058	0.0024257	0.0026323	0.0025965
AR-03 :	0.0029099	0.0026581	0.0024262	0.0023742	0.0026630	0.0025586
AR-04 :	0.0024113	0.0024245	0.0024433	0.0021487	0.0024895	0.0022711
VAR-01 :	0.0021851	0.0024145	0.0022146	0.0017045	0.0016340	0.0019979
VAR-02 :	0.0020300	0.0022190	0.0019829	0.0019316	0.0021579	0.0017090
VAR-03 :	0.0023785	0.0023913	0.0024265	0.0021771	0.0024768	0.0023914
VAR-04 :	0.0028705	0.0025000	0.0024076	0.0024330	0.0029937	0.0026569
LSTAR-01 :	0.0024420	0.0025876	0.0027490	0.0027973	0.0029573	0.0026372
LSTAR-02 :	0.0040202	0.0055192	0.0055306	0.0057352	0.0062625	0.0085641
LSTR-01 :	0.0022871	0.0025215	0.0025134	0.0024683	0.0025559	0.0026498
LSTR-02 :	0.0039812	0.0058587	0.0069575	0.0077611	0.0091252	0.0115886
TVP-01 :	0.0026682	0.0027270	0.0027765	0.0023202	0.0030999	0.0030525
TVP-02 :	0.0024245	0.0034131	0.0034650	0.0036648	0.0034072	0.0032986
TVP-03 :	0.0021676	0.0021258	0.0019596	0.0018238	0.0021900	0.0022402
PHCu-01 :	0.0019449	0.0018972	0.0018663	0.0016117	0.0017803	0.0019719
PHCu-02 :	0.0019084	0.0018319	0.0018899	0.0016865	0.0019978	0.0021553
PHCu-03 :	0.0017452	0.0018238	0.0018475	0.0017273	0.0022803	0.0016786

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