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Financial Crises and Time-Varying Risk Premia in a Small Open Economy: A Markov-Switching DSGE Model for Estonia

Boris Blagov*

Abstract

Under a currency board the central bank relinquishes control over its monetary policy and domestic interest rates converge toward the foreign rates. Nevertheless a spread between both usually remains. This spread can be persistently positive due to increased risk in the economy. This paper models that feature by building a DSGE model with a currency board, where the domestic interest rate is derived as a function of the foreign rate, the external debt position and an exogenous risk premium component. Applying Markov-Switching allows for time variation in the volatility of the risk premium component. The model shows that the size of risk premia shocks in an economy with a currency board is small in quiet times but the shocks are much larger during crises, which the standard model would understate. The model is applied with Bayesian methods to Estonian data and is able to match the banking and financial crises.

JEL Code: E32, F41, C51, C52

Keywords: Markov-Switching DSGE Models, currency board, stochastic risk premium

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Non-technical summary

This paper uses a novel modelling approach to capture the non-linearities of the risk premium on the interest rates in a small open economy with a currency board. Smaller economies often use this board mechanism as a stabilization mechanism by pegging their currency to a stronger and less volatile currency, which in theory should lead to convergence between domestic and foreign interest rates. However, a spread between the rates may emerge during rare events such as a crisis or a loss of credibility of the board, which reflects the risk associated with the system. This spread might sometimes get excessively large and this may have detrimental effects on the economy. In the case of Estonia, the 3-month interbank interest rates were up to 14% higher during the 1997–1999 banking crisis than their European counterparts.

When it comes to modelling a currency board, Dynamic Stochastic General Equilibrium (DSGE) models are highly stylised. The mechanism is usually simulated by the imposition of a fixed exchange rate, which leads to domestic and foreign interest rates becoming identical. This simple technique is unable to capture the inherent non-linearities of the board. In this paper, a standard DSGE model is extended to a Markov-Switching Rational Expectations model, where domestic rates are derived as a function of the foreign rates and an additional component. This component allows for a spread between the two rates and is composed of an endogenous variable and an exogenous process. The exogenous part reflects the risk premium. Through this extension, the model may then have two regimes, a low-risk premium (or "high-credibility") regime and a high-risk premium ("low-credibility") regime.

Estimation with real data shows that the model is well suited to accommodate the dynamics and evolution of the Estonian economy. It is able to identify the banking crisis in 1997 and the financial crisis of 2008. The advantage of this framework is that the effects of risk-premium shocks can be quantified in each regime separately and state contingent analysis can be performed. It is shown that during the low-risk premium regime, the size of interest rate shocks is negligible, while in the second regime interest rate shocks are substantial. A standard DSGE model is only able to capture a middle-ground scenario, overestimating the shocks compared to the first regime and understating them compared to the second.

Contents

1. Introduction

A key factor in a currency board mechanism is the inherent link between the interest rates of the pegged currency and the foreign rates. Under ideal conditions, these two rates should converge and eventually be identical. In practice they are not equal. While small discrepancies always arise, a multitude of factors, such as speculation against the board or a banking crisis can contribute to a persistent, positive spread. The literature on pegs largely ignores this issue. In most models, especially in the DSGE field, in the absence of money the mechanism is simulated using a short-cut by closing the exchange rate channel, which leads to a one-to-one relationship between the interest rates.[1] A discrepancy may be added by the introduction of a random disturbance (which can be interpreted as a risk-premium shock). This setup, however, is at odds with the empirical evidence. In general, many countries with a currency board have experienced speculation against the mechanism or problems in the banking sector at one point or another. This has in turn led to a an opening of the spread. During such periods the economy is naturally more sensitive to shocks and macroeconomic variables may react strongly to economic disturbances.

The standard modelling approach is unable to capture these events. The conventional approach with an interest rate identity and a random disturbance cannot distinguish between periods of large and small spreads. In turn if one studies the propagation of shocks with this model, the system would produce stronger responses during "quiet" times while underestimating the effects of disturbances in the banking sector during sensitive periods. It is probable that during the former, risk would play a negligent role, while it could have important implications in the latter.

This paper addresses that issue. It builds a model based on Gali and Monacelli (2005) with the addition of imperfect risk sharing in the spirit of Justiniano and Preston (2010). By fixing the exchange rate, the domestic interest rate can be derived as a function of its foreign counterpart and two additional components as in Gelain and Kulikov (2009). The first is an endogenous component, which can be interpreted as the net foreign asset position or "debt sensitivity". If domestic agents are heavy borrowers the spread opens and it becomes more costly to borrow further. This allows for an endogenous discrepancy between the rates. Without this feature any difference would be attributed to the risk premium. The second component, an external risk premium shock, aims to capture the exogenous part of the spread. The key feature is that by the means of Markov-Switching the volatility of the innovations is

[1] For a baseline model see Gali and Monacelli (2005).

time-varying. The agents are aware that a switch is possible and act accordingly.[2] The change itself and its probability are assumed to be exogenous to the individuals. As such, this model also presents an alternative method to Justiniano and Primiceri (2008) in dealing with time-variation in the volatility of macroeconomic shocks.

The model is then applied to Estonia which had a currency board from 1992 up to the introduction of the Euro in 2011. Throughout this period the banking system experienced several crises as well as the global financial turmoil. Figure 1 depicts the annual Estonian and European interbank rates (TALIBOR and EURIBOR) starting from 1996. The 1997–2000 banking crisis and the 2008 financial crisis are easily identified. Utilizing Bayesian methods, the model captures these peculiarities by identifying significant time-variation in the risk premium and endogenously matching the crises. The impulse responses confirm the proposed hypothesis that under a currency board, during "normal" times the risk premium shocks play a minor role but have a major impact during "special" periods, while a standard model would produce "in-between" results.

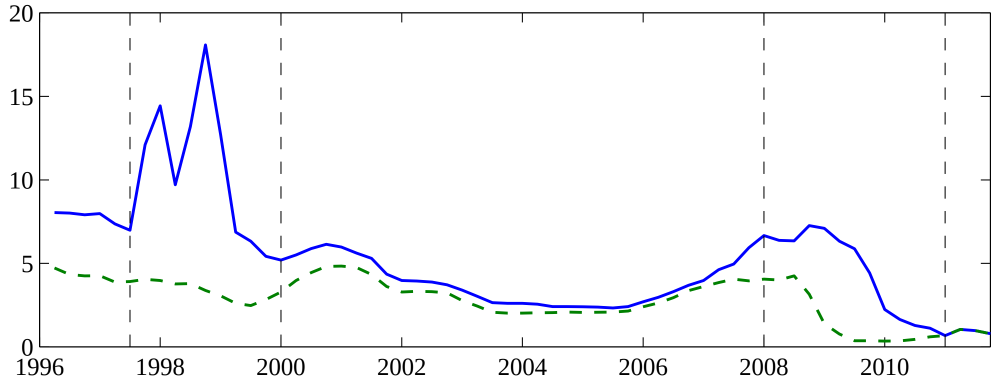

Figure 1: Annual interbank interest rates in %, Estonia's TALIBOR (—) and EURIBOR (- -)

2. A Model to Capture the Spread

Estonia is a rapidly growing small open economy (SOE) that is heavily involved in trade. Exports and imports for the last decade have been over 50%

[2]Due to the linearization up to first order, this additional feature does not influence the individual decision making because the switching is incorporated in the variance of the exogenous shock.

of GDP each.[3] Since 1992 a currency board has been in place. It pegged the exchange rate first to the German Deutsche Mark and then to the Euro. Under the mechanism, the central bank forfeits its control over conventional monetary policy instruments such as steering the interest rates or controlling the money supply directly. Hence it cannot function as a *lender of last resort* neither to the government, nor to banks. As a result, the domestic currency inherits the stability of the one it is pegged to and domestic interbank interest rates should follow the interest rates of the foreign economy. Any spread between the two would entail the risk associated with the domestic economy.

The goal of this section is to outline a model that might be suitable to describe such an economy. As a basis is taken the work of Justiniano and Preston (2010) — a SOE a lá Monacelli (2005) with incomplete markets, hybrid inflation dynamics, habit formation and a handful of structural shocks. To accommodate the currency board feature, the model is closed via one of the methods outlined in Schmitt-Grohe and Uribe (2003), namely through the exchange rate channel. Hence the nominal interest rate becomes a function of the foreign interest, net foreign asset position and a risk premium, which is a feature also adopted in the Estonian DSGE model of Gelain and Kulikov (2009).

On the demand side, consumers maximize utility by choosing the optimal allocation of consumption and labour, subject to a budget constraint.

$$E_0 \sum_{t=0}^{\infty} \beta^t \vartheta_t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right] \tag{1}$$

Their utility is subject to preference shocks ϑ_t. β is the discount factor, σ is the inverse of the elasticity of substitution as well as the risk-aversion coefficient, N_t denotes labour and C_t denotes consumption. The latter is a bundle of domestic and foreign goods and, for simplicity, the habit formation of the original model is omitted.

Income is the total sum of the wages obtained by working as an employee $W_t N_t$, the profits of import $\Pi_{F,t}$ and export firms $\Pi_{H,t}$ (which are owned by the individuals) and the returns on domestic and foreign bonds. The formal budget constraint is

$$P_t C_t + B_t + e_t B_t^* = B_{t-1}(1+i_{t-1}) + e_t B_{t-1}^*(1+i_{t-1}^*)\Phi(D_t, \phi_t) + W_t N_t + T_t \tag{2}$$

As standard in the open-economy literature, an asterisk (*) denotes foreign variables. e_t is the exchange rate, B_t are the net stocks of one-period Arrow-Debrew securities with nominal interest i_t. The CPI is denoted by P_t and T_t

[3] Source: Eurostat.

stands for taxes. $\Phi(D_t, \phi_t)$ is a debt elastic interest rate premium with

$$\Phi_t = e^{-\chi(D_t+\phi_t)}$$

D_t is the real quantity of the consumer's net foreign asset position in relation to steady state output $\bar{Y}$

$$D_t = \frac{\mathcal{E}_t B^*_{t-1}}{\bar{Y} P_{t-1}}$$

$\mathcal{E}_t$ is the bi-lateral nominal exchange rate. $\Phi(\bullet)$ follows Benigno (2001), Schmitt-Grohe and Uribe (2003) and Gelain and Kulikov (2009). The intuition behind the first component, D, is that domestic agents face a cost when participating in world markets. As lenders ($D_t > 0$, accumulation of foreign debt), the households receive a lower remuneration than the market rate. As borrowers ($D_t < 0$) they pay a premium over the interest rate. The premium is sensitive with respect to the net foreign asset position (measured by χ). ϕ is a risk-premium shock that captures exogenous forces outside of the model that introduce risk to the system.

In the context of the SOE literature (Kollmann (2002), Schmitt-Grohe and Uribe (2003), Justiniano and Preston (2010)), through the incomplete asset markets assumption, ϕ can be interpreted as an uncovered interest rate parity (UIP) shock and D functions as deviation from the parity — a positive or negative premium depending on the domestic country being a lender or borrower in the world market.[4] In this setting, however, after closing the model, the UIP equation will appear as a (passive) monetary policy rule where the domestic interest rate i is a function of the foreign interest rate i^*, the debt position D and the risk-premium shock ϕ. Thus, the domestic interest rate would be increasing in the level of household borrowing from the world economy and would also be subject to exogenous shocks.

Log-linearization of the first order conditions leads to the usual Euler equation and leisure-consumption trade-off. The consumption dynamics is represented by the conventional inter-temporal equation with lower-case letters denoting logs of the variables and π being the inflation rate:

$$c_t = E_t\{c_{t+1}\} + \frac{1}{\sigma}(E_t\{\pi_{t+1}\} - i_t) + \frac{1}{\sigma}(1 - \rho_\vartheta)\vartheta_t \tag{3}$$

On the producer side, the firms employ labour and maximize profits subject to the costs and demand. They either produce goods for domestic consumption and exports or import goods from abroad. The "Law of one price" fails to hold for imported goods and both type of firms have some market power. Price is determined in a hybrid manner — in every period a part sets their price based

[4] See Benigno (2001), p. 12.

on expectations about future demand and costs a lá Calvo (1983) and the rest use past information. Hence the inflation dynamics of both goods is forward- and backward-looking.

$$(1+\beta\delta_H)\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \delta_H \pi_{H,t-1} + \lambda_H mc_t + \mu_{H,t} \quad (4)$$

$$(1+\beta\delta_F)\pi_{F,t} = \beta E_t\{\pi_{F,t+1}\} + \delta_F \pi_{F,t-1} + \lambda_F \psi_t + \mu_{F,t} \quad (5)$$

Here δ_H and δ_F denote the respective fractions of firms that use past indexation and ψ_t is the deviation from the "Law of one price". It presents the difference between the prices of the imported good at home and at the world markets in logs: $\psi_t \equiv e_t + p_t^* - p_{F,t}$. Both λ's are functions of the parameters of the model and prices may be subject to cost-push shocks $\mu_{H,t}$ and $\mu_{F,t}$. Marginal costs are denoted by mc_t. They are a function of domestic and foreign output (y_t and y_t^*), the terms of trade s_t, consumption c_t and technology a_t.

The CPI is a mixture of domestic and foreign prices weighted by their respective share, which is the openness of the economy (α). Hence the CPI inflation is

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t} \quad (6)$$

The third block of the model addresses the openness of the economy. Terms of trade are defined as the relation between the price of foreign goods $P_{F,t}$ to the price of home goods $P_{H,t}$ or in logs: $s_t = p_{F,t} - p_{H,t}$. The bilateral real exchange rate q_t is the ratio between the CPIs of the two countries expressed in domestic currency, which gives rise to the following relationship up to first order

$$q_t = e_t + p_t^* - p_t(1-\alpha)s_t \quad (7)$$

The law of one price is not assumed to hold, which translates into a discrepancy between world prices of world goods and prices of world goods in the domestic economy ($\Psi_t = \dfrac{\mathcal{E}P_t^*}{P_{F,t}} \neq 1 \Leftrightarrow \psi_t \neq 0$). Therefore the real exchange rate is a function of the difference

$$q_t = e_t + p_t^* - p_t = \psi_t + (1-\alpha)s_t \quad (8)$$

and the dynamics of the nominal exchange rate under a flexible exchange rate regime are

$$\triangle e_t = q_t - q_{t-1} + \pi_t - \pi_t^* \quad (9)$$

This leads to a UIP condition under incomplete asset markets in the form

$$(i_t - E_t\{\pi_{t+1}\}) - (i_t^* - E_t\{\pi_{t+1}^*\}) = E_t\{q_{t+1}\} - q_t - \chi d_t - \phi_t \quad (10)$$

where d_t is a log-linear approximation of D_t around the steady state.[5] Under flexible exchange rates, the model would be closed by choosing a monetary policy rule that pins down the interest rate i. Then d_t and ϕ_t could be interpreted as deviations from the uncovered interest rate parity.[6] As a net borrower, the consumer will face a positive spread between the interest rates (adjusted for inflation expectations). In the absence of money, the introduction of an exchange rate peg is a short cut to mimic the implications of a currency board.[7]

$$\triangle e_t = 0 \tag{11}$$

Substituting through the exchange rate dynamics (9) in the UIP equation (10) the nominal interest rate becomes completely endogenous and a function of the foreign interest rate, the risk premium and the (real) net foreign asset position. Thus, the uncovered interest rate parity equation acts as a passive monetary policy rule.[8]

$$i_t = i_t^* - \chi d_t - \phi_t \tag{12}$$

Here, the risk premium component gives desirable characteristics to the formation of domestic interest rates. The debt sensitivity χ is assumed to be strictly positive. If domestic households are net borrowers on the world financial market, they have to pay a premium on the world interest rate (as d_t is negative). This translates into higher domestic interest rates i_t under a currency board, as the world interest rate i^* is exogenous to the home country. Also, i_t may be subject to an exogenous shock which can be interpreted as a risk-premium. Hence, the country could experience, on one side, a bigger spread due to the households being in debt, or on the other, this spread may come as an exogenous force such as a country specific shock or an interest rate related crisis.

The final equation needed is the evolution of the net foreign asset position, which may be interpreted as the current account.[9], p.13. It is given by

$$d_t - \frac{1}{\beta} d_{t-1} + = y_t - c_t - \alpha(q_t + \alpha s_t) \tag{13}$$

The rest of the world is also modelled differently from Justiniano and Preston (2010) and follows Chen and Macdonald (2012) and Lubik and Schorfheide

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[5] In the steady state the debt is assumed to be zero.

[6] See Benigno (2001), p. 12.

[7] Gali and Monacelli (2002), p.17.

[8] "Passive" is not used in the sense of a less-aggressive monetary policy (coefficient for inflation in the interest rate rule < 1) but rather in its general meaning. The precise rule in this case is $\triangle e_t = 0$.

[9] See Benigno (2001).

(2007) — not as a VAR(p) process but as an AR(p) processes, which might relax some potentially tight cross-equation restrictions.[10] Thus, in total there are eight exogenous variables, modelled by AR(1) processes that are governed by eight structural shocks: ε_t^a technology, ε_t^ϑ preferences, $\varepsilon_t^{\mu_H}$ domestic cost-push shock, ε^{μ_F} import cost push shock, ε_t^ϕ risk-premium shock, $\varepsilon_t^{y^*}$ world output shock $\varepsilon_t^{\pi^*}$ world cost-push shock and $\varepsilon_t^{i^*}$ world monetary policy shock.[11]

$$\begin{aligned}
a_t &= \rho_a a_{t-1} + \varepsilon_t^a && \text{with} \quad \varepsilon_t^a \sim N(0,\sigma_a^2) && (14)\\
\vartheta_t &= \rho_\vartheta \vartheta_{t-1} + \varepsilon_t^\vartheta && \text{with} \quad \varepsilon_t^\vartheta \sim N(0,\sigma_\vartheta^2) && (15)\\
\mu_{H,t} &= \rho_{\mu_H} \mu_{H,t-1} + \varepsilon_t^{\mu_H} && \text{with} \quad \varepsilon_t^{\mu_H} \sim N(0,\sigma_{\mu_H}^2) && (16)\\
\mu_{F,t} &= \rho_{\mu_F} \mu_{F,t-1} + \varepsilon_t^{\mu_F} && \text{with} \quad \varepsilon_t^{\mu_F} \sim N(0,\sigma_{\mu_F}^2) && (17)\\
\phi_t &= \rho_\phi \phi_{t-1} + \varepsilon_t^\phi && \text{with} \quad \varepsilon_t^\phi \sim N(0,\sigma_\phi^2) && (18)\\
y_t^* &= c_{y^*} y_{t-1}^* + \varepsilon_t^{y^*} && \text{with} \quad \varepsilon_t^{y^*} \sim N(0,\sigma_{y^*}^2) && (19)\\
\pi_t^* &= c_{\pi^*} \pi_{t-1}^* + \varepsilon_t^{\pi^*} && \text{with} \quad \varepsilon_t^{\pi^*} \sim N(0,\sigma_{\pi^*}^2) && (20)\\
i_t^* &= c_{i^*} i_{t-1}^* + \varepsilon_t^{i^*} && \text{with} \quad \varepsilon_t^{i^*} \sim N(0,\sigma_{i^*}^2) && (21)
\end{aligned}$$

Markov Switching Extension

The main block consists of two models — the prototypical DSGE version ($\mathcal{M}_1$) and a Markov-Switching scenario ($\mathcal{M}_2$). In the robustness section 5 further MS models are evaluated. Two states of the economy are allowed in the main model. The transition probabilities of the regimes are represented by the matrix P:

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

where p_{ij}, the $\{i,j\}$ element of the matrix, is the transition probability from state i to state j, $Prob(s_{t+1} = j|s_t = i)$.[12] The different states are characterized by time-varying stochastic volatility of the risk-premium (18)

$$\phi_t = \rho_\phi \phi_{t-1} + \varepsilon_t^\phi \quad \text{with} \quad \varepsilon_t^\phi \sim N(0,\sigma_\phi^2(s_t)) \tag{22}$$

As such, the model attempts to capture the heteroskedasticity of the risk-premia during financial crises.

[10] Chen and Macdonald (2012), p. 1095.

[11] A complete list of the equations can be found in the Appendix 1.

[12] The literature does not follow a single convention. Here the notation in Cho (2011) is adopted. Hamilton (1989) or Kim and Nelson (1999) use $p_{ij} = Prob(s_{t+1} = i|s_t = j)$, so that p_{21} is the probability of moving from state 1 to state 2.

Solution Methodology

The equations may be cast in the following state-space form:

$$B_1(s_t)X_t = E_t\{A_1(s_t, s_{t+1})X_{t+1}\} + B_2(s_t)X_{t-1} + C_1(s_t)Z_t \tag{23}$$

$$Z_t = R(s_t)Z_{t-1} + \epsilon_t \quad \text{with} \quad \epsilon_t \sim N(0, \Sigma^2(s_t)) \tag{24}$$

A large part of the MS-DSGE papers are devoted to the technical aspects of solving the system: in contrast to non-switching models, stability in the first moments does not imply stability in the second moments.[13] As an example, both states may exist and be unique in a standard model but if the Markov-Switching system jumps too often between them it could become unstable. The methods of Linear Rational Expectations (LRE) models cannot be directly applied. The available solution concepts are mainly centred around stability in the second moments. Davig and Leeper (2007) show that if the shocks are bounded in variance, there can be a stable solution to the Markov-Switching system. Farmer et al. (2010) and Cho (2011) deal with the concept of mean-square stability with unbounded shocks. The algorithm of Farmer et al. (2010) utilizes the idea of a MSV solution (Minimum State Variable Solution) in the sense of McCallum (1983) and is able to find all possible solutions to the system but no clear rule at how to choose between them. Cho (2011) on the other hand derives also conditions for determinacy and provides rationale why only one of the MSV solutions is relevant to the economic problems at hand by introducing the so-called "non-bubble condition". Therefore, this paper adopts the solution strategy of Cho (2011).

An important remark is that the model discussed in this paper is technically not a "proper" Markov-Switching Rational Expectations model (MSRE). The switching component is in the stochastic volatility and up to first order the two states are identical for both regimes. This leads to an appealing property — the steady state is the same for each regime and would coincide with the standard Rational Expectation case. Thus, issues which are not yet clear such as how does the economy jump from one steady state to another do not arise. A more detailed discussion of the solution method can be found in the Appendix.

Estimation Procedure

The model is estimated using Bayesian methods. A prior is put on the parameters and is combined with the likelihood function to form the posterior, which is minimized and simulated through a Markov-Chain Monte Carlo

[13] Cho (2011), p.2.

(MCMC) algorithm. However, in contrast to the standard DSGE case, MSRE models are not so straightforward to estimate. The issue lies in the evaluation of the likelihood which is dependent on the history of the states.

Due to the Markov-Switching nature of the model, the Kalman filter cannot be applied since the possible paths grow exponentially with the number of observations. Even just ten observations of a two-state economy give 1024 possible paths. To solve this problem, the literature usually takes two approaches. One proposition is to construct the likelihood function through Gibbs sampling.[14] The other is to use Kim's Filter — a combination between Kalman and Hamilton filters where at each step, only a fixed number of the future time periods is evaluated and then collapsed to a finite and small number of paths.[15] This keeps the computation of the likelihood tractable. The posterior is then evaluated using Bayes' rule conditional on the states and the probabilities

$$p(\theta, P, S|Y) = \frac{p(Y\,|\theta, P, S)\, p(S\,|P)\, p(\theta, P)}{\displaystyle\int p(Y\,|\theta, P, S)\, p(S\,|P)\, p(P, \theta)\, d(\theta, P, S)} \tag{25}$$

This paper uses Kim's filter and the posterior is evaluated using a Metropolis-Hastings (MH) algorithm. Kim and Nelson (1999) suggest an optimal history of $2^{(k-1)}$ time periods, where k is the number of states in the system. Therefore, the history of S_t and S_{t-1} is preserved and four states are carried through on each iteration.

Farmer et al. (2009) point out that with regime switching the posterior could be highly non-Gaussian and the mean values of this distribution may actually lie in a region where the support is flat. Hence, it is of crucial interest to find the posterior mode. This task, however, may be computationally intensive, as the posterior is often multi-modal and the optimization may get stuck at a local mode. Farmer et al. (2009) propose a specific block-wise optimization algorithm to deal with the problem, while Sargent et al. (2009) use a combination between a Gibbs sampling version of Chris Sims' CSMINWEL routine, followed by a combination of the BFGS Quasi-Newton algorithm and the Fortran IMSL routine. Nevertheless, Liu and Mumtaz (2011) and Chen and Macdonald (2012) report successful usage of CSMINWEL alone. For the current estimation, Sims' routine faced particular difficulties finding the global mode and often got stuck at local maxima with a high likelihood value (the procedure is a minimization algorithm, so high values are undesirable). For the maximization of the likelihood the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) has been employed and particularly its extension for

[14] As in Bianchi (2012) or Kim and Nelson (1999), ch. 9.

[15] See Kim and Nelson (1999), ch. 5.

DSGE models by Martin Andreasen.[16] The function is based on the Evolution Strategy algorithms and its strengths lie in the fact that it does not calculate gradients or approximate numerical derivatives and therefore it has an advantage when the functions have discontinuities, ridges, or local optima.[17]

3. Taking the Model to Data

In total there are eleven endogenous and eight exogenous variables. The endogenous vector X_t consists of $[c, y, i, q, s, \psi, \pi, \pi_H, \pi_F, d, mc]'$ and the vector of exogenous variables Z_t constitutes of $[a, \vartheta, \mu_H, \mu_F, \phi, y^*, \pi^*, i^*]'$. The solution to the state-space form (23) is the transitional equation used in the initial step of Kim's filter in conjunction with a measurement equation

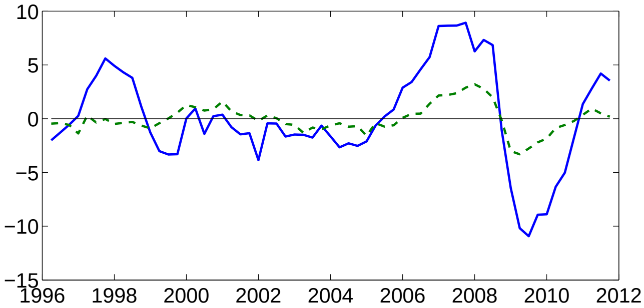
$$X_t = \Omega^*(s_t)X_{t-1} + \Gamma^*(s_t)Z_t \tag{26}$$

$$Y_t = HX_t + Q_t \tag{27}$$

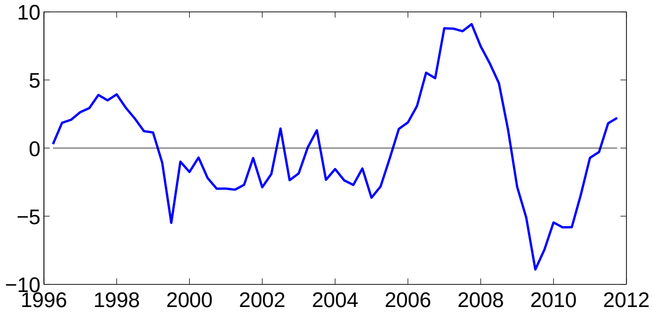
There are eight structural shocks that allow for up to eight variables to be present in the likelihood function without the need to introduce measurement errors. The vector of observables Y_t consists of: Estonian output growth, consumption, inflation and nominal interest rate, European output, inflation and interest rate. The "Rest of the World" is modelled with EU27 data as the majority of Estonia's trading partners are European countries. Moreover, the exchange rate was pegged first to the German Deutsche Mark and then to the Euro. Output is the series of GDP divided by the active population and consumption is also scaled per capita. Labour is employment among the individuals from 16 to 64 years of age. Inflation rate is quarterly HICP data. The nominal interest rate for Estonia is TALIBOR and for Europe — EURIBOR (three-month money market rate, converted to quarterly frequency).

The series are expressed as percentage deviations from trend, where the detrending has been carried out by the HP filter with a lambda of 1600, a standard in the literature. All data have been seasonally adjusted and logs were taken where appropriate. Estimation uses quarterly data from the period of 1996Q1 to 2011Q4, where the actual sample size starts from 1996Q2 (63 observations in total). This time span covers the existence of TALIBOR, which has been introduced in 1996. Figure 2 displays the series used in the model.

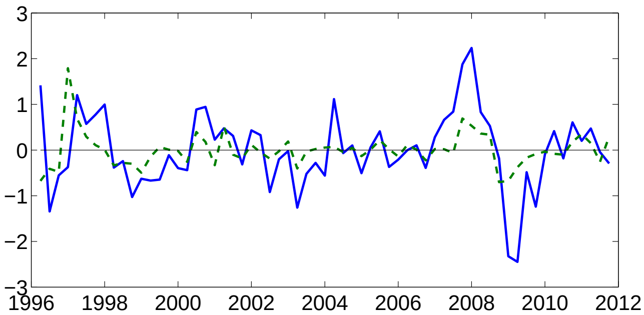
[16]See Andreasen (2008).
[17]See Hansen (2006).



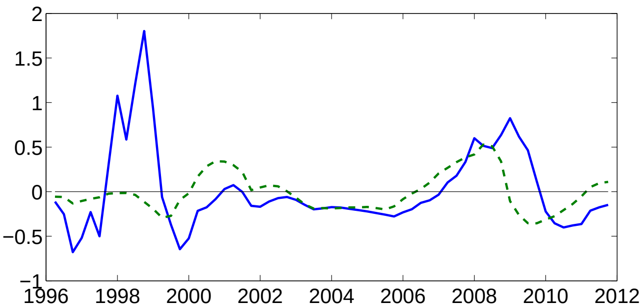
(a) RGDP per capita



(b) Consumption



(c) Inflation



(d) TALIBOR and EURIBOR

Figure 2: De-trended and seasonally adjusted quarterly data
Note: Blue line (—) is Estonian data, dashed green line (- -) is European data. In %.

In the DSGE literature, that the interest rate is often either taken “as is” or as demeaned series without any detrending whatsoever. There are several reasons why a detrending filter has been applied here. From an econometric perspective, while TALBIOR does not exhibit non-stationarity, EURIBOR cannot reject a unit root without the inclusion of a trend. Theoretically, the model is a log-linearization around a steady state. If there is a publicly adopted target, such as the 2% of the European Central Bank and the series are fluctuating around it, it is plausible to demean the data or subtract the target (adopt it as a steady state). Here, it is a special case as the adoption of a currency board is typically done to stabilize prices, inflation and the interest rates which often produces a downward trend. Figure 2 depicts the interest rate with HP and linear filter.

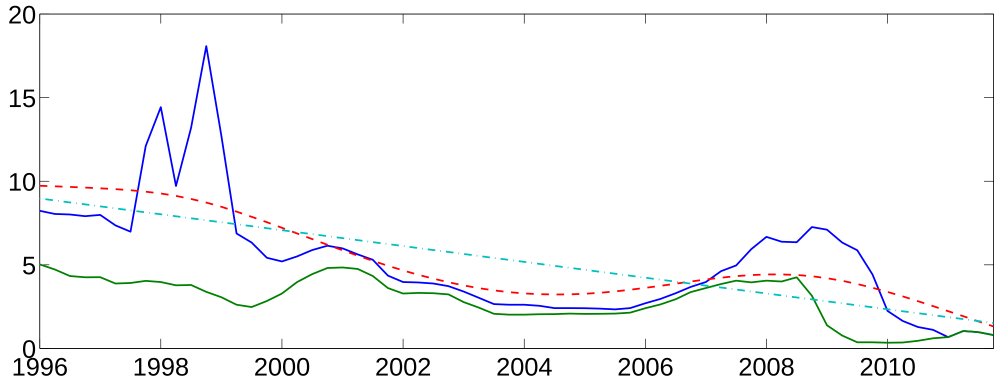


Figure 3: Linear (cyan) and HP-filtered (red) trend of the interest rate series

Due to the controversial nature of detrending, or in this case — not-detrending, several models are estimated. The main model uses an HP-filter to detrend the series (a linear trend has also been explored), while in the robustness section a model without detrending is presented. Throughout all specifications, the main findings are robust.

3.1. Priors

Bayesian estimation is always sensitive to the choice of priors. There are not many microeconomic studies estimating deep parameters such as discount factor or elasticity of substitution for Estonia. Gelain and Kulikov (2009) estimate a medium-sized DSGE model for the country and take most priors from Smets and Wouters (2003). Here most values are either from Gelain and Kulikov (2009) or are standard values from the literature. Table 1 summarizes the parameter moments.

Table 1: Prior distributions and basic moments ("PM" indicates Point-Mass)

	Dist.	Mean	Std.Dev.		Dist.	Mean	Std.Dev.
p_{11}	$Beta$	0.9	0.1	ρ_{μ_H}	$Beta$	0.7	0.1
p_{22}	$Beta$	0.9	0.1	ρ_{ν}	$Beta$	0.7	0.1
β	PM	0.995	—	ρ_{ϕ}	$Beta$	0.7	0.1
φ	Gam	2	0.25	c_{y^*}	$Beta$	0.85	0.1
θ_H	$Beta$	0.75	0.1	c_{π^*}	$Beta$	0.85	0.1
θ_F	$Beta$	0.5	0.1	c_{i^*}	$Beta$	0.85	0.1
α	PM	0.5	—	σ_{μ_F}	$IGam$	1	∞
σ	Gam	1	1	σ_{μ_H}	$IGam$	1	∞
η	Gam	2	0.25	σ_a	$IGam$	1	∞
δ_H	$Beta$	0.5	0.15	σ_{ν}	$IGam$	1	∞
δ_F	$Beta$	0.5	0.15	σ_{ϕ}	$IGam$	1	∞
χ	Gam	0.01	0.01	σ_{y^*}	$IGam$	1	∞
ρ_a	$Beta$	0.7	0.1	σ_{π^*}	$IGam$	1	∞
ρ_{μ_F}	$Beta$	0.7	0.1	σ_{i^*}	$IGam$	1	∞

The discount factor β is fixed at 0.995 implying an annual run interest rate of about 4% and the coefficient of openness α is set at 0.5. The inverse elasticity of substitution is set to unity. Frisch elasticity of labour supply φ, as well as the elasticity between home and foreign goods η have means of 2 and a standard deviation of 0.25 following Gelain and Kulikov (2009). The Calvo parameter for prices θ is chosen to be 0.75, providing an average duration of price contracts of a year. The share of forward and backward-looking firms, as well as the debt elasticity χ are taken from Justiniano and Preston (2010). The autoregressive coefficients for Europe are 0.85 each as in Smets and Wouters (2003) and similarly the priors for the Estonian economy have been chosen all to equal 0.7. All shocks are of size 1 with unbounded variance with the exception of the risk-premium.

4. Estimation Results

First, the model is solved using the initial parameters and the likelihood is approximated by Kalman's ($\mathcal{M}_1$) or Kim's filter ($\mathcal{M}_2$). The posterior density is then minimized to find the posterior mode using the CMA-ES algorithm. Cut-off criterion for the minimum is 10^{-16}. Since the evolutionary algorithm uses the Variance-Covariance matrix and evaluates a random number of possible paths at each point it is less dependent on initial values. In this case it always converged to the same mode.

Once a minimum is obtained, a MCMC procedure is initiated with the inverse Hessian estimated at the posterior mode. In total, 4 blocks of 375 000 draws are estimated, with the first 75 000 discarded and every 30-th draw afterwards saved for a volume of 10 000 observations per block. The constant is tuned to attain an acceptance ratio of roughly 20%. In all cases the procedure converged to the same mean. Table 2 shows the estimated coefficients for both models.

4.1. $\mathcal{M}_1$: A standard DSGE model for Estonia

Most parameters for the no-switching scenario are a suitable representation of a small economy involved heavily in trade. The elasticity of substitution between foreign and home goods η is 2.366 and the whole posterior density is shifted to the right of the prior suggesting a good integration between Estonian and European markets.[18] The risk-aversion/inverse elasticity of substitution parameter σ is about 2.4 which is a standard value in the absence of capital.[19] Indeed Gelain and Kulikov (2009) report a value of 1.33 and they do accommodate for capital formation. There is no labour data used in the model, therefore the inverse elasticity of substitution of labour φ is not identified. The posterior is closely overlapping the prior with a mean of 2.

In contrast to the elasticities, one of the coefficients in the Phillips curve is rather peculiar. While the duration of price contracts in Europe is estimated around three quarters with $\theta_F = 0.631$, the degree of price stickiness in Estonia is extremely high: $\theta_H = 0.9$. This is surprising, considering the economy is regarded as highly competitive. Prices in Estonia are estimated to be more flexible than in Europe, thus $\theta_H < \theta_F$ is expected.[20] Having a posterior away from the prior does not always guarantee that the parameter is well identified and this is the most likely cause here — θ_H might be weakly identified. The model is estimated with the series for HICP — a composite of home and foreign prices (π_H and π_F) and it is up to this data to provide identification for both θ_H and θ_F. Estonia is a country with high exports and imports and foreign goods are a large portion of the consumer basket. To counter this problem several strategies have been employed with mixed results. Using the GDP deflator series or estimating the model with the terms of trade (defined as p_F/p_H) or the real exchange rate did not lead to any significant improvement. While this coefficient introduces longer price contracts, it should not pose a problem as long as inflation at a whole is well accounted for. In fact, the model matches the volatility of inflation well (see Section 4.2).

[18]Figure 8 in the Appendix depicts the prior and posterior densities of $\mathcal{M}_1$.
[19]See Justiniano and Preston (2010), p. 104.
[20]See Randveer and Dabušinskas (2006) and Schwab (2011) among others.

The autoregressive coefficients are similar to those reported in Gelain and Kulikov (2009) where the prior plays a more-important role and the persistence of almost all shocks is about 0.6 to 0.7. The data seem informative about the volatility of all exogenous variables with the exception of technology, as shown in Figure 8 in the Appendix. Notably, the volatility of the Estonian structural shocks is higher than the European ones, which is a standard feature for small rapidly growing economies. The volatility of the risk-premium σ_ϕ is estimated at 0.472 and the debt elasticity $\chi \sim 0.028$. The latter coincides with the value of Gelain and Kulikov (2009), who report a risk premium sensitivity of 0.029. The standard deviation of the risk-premium is 0.5, which is also a plausible result. This is quarterly data, hence up to 2% of the volatility of the interest rate may be attributed to the risk premium. The model scores a marginal data density of –430.723, which has been estimated using the Modified Harmonic Mean (MHM) estimator.

Convergence is assessed using both formal tests and graphical methods, following An and Schorfheide (2007). Section A.3 in the Appendix contains the figures and tables of the first model. Figure 9 plots the recursive means for all parameters, while figure 10 shows the trace plots. Most parameters seem to converge within 5000 draws. Actually only two of the 24 parameters do not seem to experience smooth convergence and they are both related to the volatility of shocks, which is not detrimental to the main results of the model — technology σ_a and preference σ_ν. This can be observed by their trace and recursive means plots and the autocorrelation estimates. Table 6 in the Appendix shows the Lewis-Raferty diagnostics (1.612) and the autocorrelation among the draws. The latter dies out by the 10-th lag for all but the technology and preference shock again. Having presented the standard DSGE model, it is time to explore the regime switching specification.

Table 2: Estimated coefficients at the posterior mean. $\mathcal{M}_1$: Model with fixed parameters,$\mathcal{M}_2$: MS Model. $\mathbb{M}$ denotes the marginal data density, 5% and 95% percentiles in brackets

	Distribution	Prior Mean	$\mathcal{M}_1$	$\mathcal{M}_2 : S_t = 1$	$\mathcal{M}_2 : S_t = 2$
p_{11}	$Beta$	0.900	—	0.936 [0.862, 0.984]	—
p_{22}	$Beta$	0.900	—	0.942 [0.852, 0.993]	—
β	PM	0.995	0.995	0.995	—
φ	$Gamma$	2.000	1.985 [1.608, 2.404]	1.982 [1.598, 2.399]	—
θ_H	$Beta$	0.750	0.910 [0.880, 0.938]	0.912 [0.882, 0.939]	—
θ_F	$Beta$	0.500	0.631 [0.544, 0.717]	0.645 [0.556, 0.733]	—
α	PM	0.500	0.500	0.500	—
σ	$Gamma$	1.000	2.339 [1.371, 3.694]	2.424 [1.434, 3.800]	—
η	$Gamma$	2.000	2.366 [2.011, 2.760]	2.411 [2.062, 2.781]	—
δ_H	$Beta$	0.500	0.215 [0.094, 0.371]	0.217 [0.096, 0.369]	—
δ_F	$Beta$	0.500	0.590 [0.386, 0.786]	0.594 [0.395, 0.788]	—
χ	$Gamma$	0.010	0.028 [0.014, 0.043]	0.017 [0.006, 0.029]	—
ρ_a	$Beta$	0.700	0.698 [0.520, 0.851]	0.703 [0.526, 0.854]	—
ρ_{μ_F}	$Beta$	0.700	0.700 [0.531, 0.847]	0.708 [0.537, 0.853]	—
ρ_{μ_H}	$Beta$	0.700	0.650 [0.480, 0.807]	0.670 [0.499, 0.821]	—
ρ_ν	$Beta$	0.700	0.697 [0.531, 0.842]	0.695 [0.523, 0.841]	—
ρ_ϕ	$Beta$	0.700	0.640 [0.474, 0.789]	0.646 [0.487, 0.792]	—
c_{y^*}	$Beta$	0.850	0.884 [0.790, 0.968]	0.883 [0.786, 0.969]	—
c_{π^*}	$Beta$	0.850	0.545 [0.379, 0.722]	0.548 [0.382, 0.722]	—
c_{i^*}	$Beta$	0.850	0.861 [0.783, 0.933]	0.857 [0.782, 0.925]	—
σ_{μ_F}	$IGamma$	1.000	1.225 [0.857, 1.673]	1.159 [0.799, 1.618]	—
σ_{μ_H}	$IGamma$	1.000	0.458 [0.326, 0.620]	0.425 [0.298, 0.586]	—
σ_a	$IGamma$	1.000	0.855 [0.209, 2.358]	1.060 [0.205, 3.474]	—
σ_ν	$IGamma$	1.000	11.265 [7.040, 17.150]	11.438 [7.154, 17.458]	—
σ_ϕ	$IGamma$	0.800	0.472 [0.406, 0.548]	0.119 [0.090, 0.156]	0.665 [0.533, 0.831]
σ_{y^*}	$IGamma$	1.000	0.684 [0.592, 0.791]	0.685 [0.593, 0.795]	—
σ_{π^*}	$IGamma$	1.000	0.375 [0.321, 0.438]	0.376 [0.321, 0.440]	—
σ_{i^*}	$IGamma$	1.000	0.100 [0.086, 0.116]	0.100 [0.086, 0.116]	—
$\mathbb{M}$:			-430.723	-405.5175	

4.2. $\mathcal{M}_2$: Markov-switching case

In this model, the economy may jump between two different regimes. The transition follows a Markov-Switching process with a probability matrix P. The two regimes essentially allow for heteroskedasticity of the risk premium.[21] Equations (12) and (18) (18) are replicated here:

$$i_t = i_t^* - \chi d_t - \phi_t$$

$$\phi_t = \rho_\phi \phi_{t-1} + \varepsilon_t^\phi \quad \text{with} \quad \varepsilon_t^\phi \sim N(0, \sigma_\phi^2(s_t))$$

There are now 3 more parameters to be estimated: p_{11}, p_{22} and $\sigma_\phi(s_t)$. In contrast to $\mathcal{M}_1$, the likelihood value is obtained by Kim's filter, which is an approximation over Kalman's filter. Most of the parameters are extremely close to the non-switching specification with largely overlapping distributions.[22] Differences are usually in the second or third digit after the decimal with a few exceptions such as the technology shock parameter.

The maximum likelihood procedure estimates two significantly different coefficients for the risk-premium volatility: $\sigma_\phi(low) = 0.119$ and $\sigma_\phi(high) = 0.665$. Neither of them is overlapping with the value from $\mathcal{M}_1$ of 0.472 [0.406, 0.548], which appears to be a "compromise" between the two. The high volatility, $\sigma_\phi(high) = 0.665$, translates into a standard deviation of almost 2.5% of the risk-premium component annually. Meanwhile, the uncertainty around the low-volatility state is much less than 25 basis points which is a good value for non-crisis periods. Central banks usually meet every quarter and, if at all, adjust the central bank interest rate with 0.25%. Furthermore, if the change is expected it does not contribute to the unexpected component at all.

The probability of being in the high-volatility state is 0.942. This corresponds to roughly 17 time periods or about 4 years on average. The actual probability of which state the economy was in at each point in time is estimated through Hamilton's filter. Kim's smoothing algorithm is then employed recursively to take into account the complete history. The bottom graph of figure 4 depicts smoothed and not smoothed probability of being in the second regime. It clearly shows the time-variation in the volatility of the risk premium on TALIBOR. Most likely ($p \sim 1$) the *high* state prevailed from the second quarter of 1997 to about the beginning of 2001 and then again for a year during the financial crisis.

[21] See section 5.1 for a model where more parameters are allowed to switch simultaneously.

[22] Distribution figures and convergence diagnostics for this specification may be found in section A.4 of the Appendix.

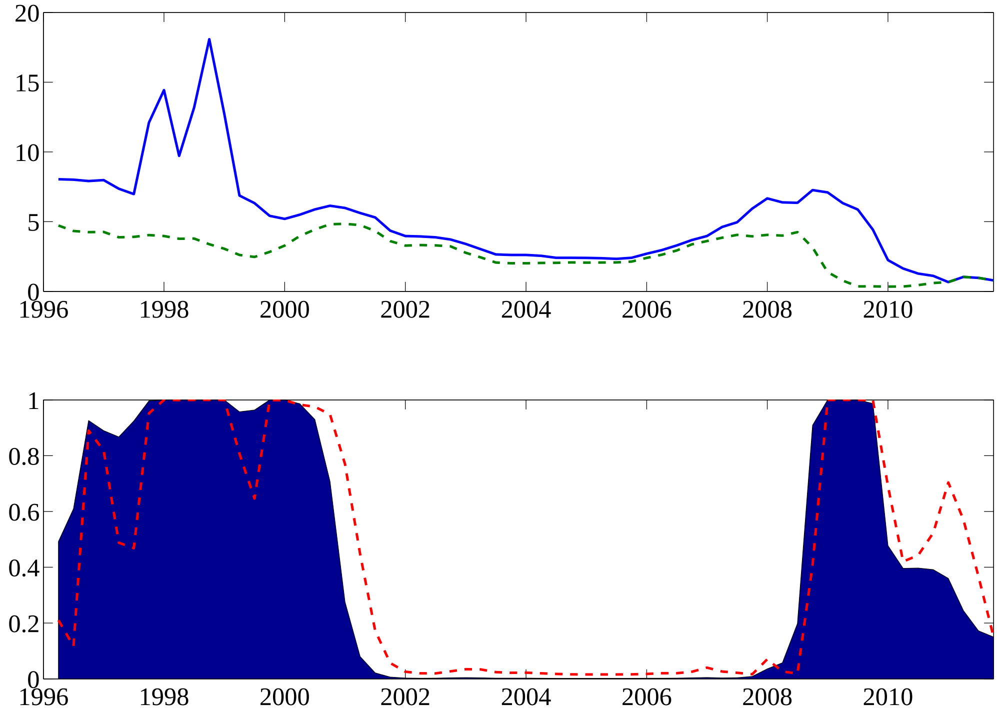


Figure 4: Top: Annualized 3-month interbank interest rates, TALIBOR (—) and EURIBOR (- -)
Bottom: Smoothed (⊓) and not-smoothed (- -) probability of $\sigma_\phi(s_t) = \sigma_\phi(high)$.

The model captures the dynamics of the Estonian interbank interest rate quite well. The two spreads between the TALIBOR and EURIBOR stand for several important events — a banking crisis and the financial turmoil. The spread between the second quarter of 1997 (when the probability peaks to one) and 2001 was marked by the Asian and Russian crises and a following banking crisis, which strained the currency board. The former affected investor confidence as the Estonian TALSE index lost over 60% of its value in the third quarter of 1997. Speculations against the currency board emerged and the banking system was put under pressure.[23] The onset of the Russian crisis dealt a huge blow to an already weakened economy. The sharp devaluation of the rouble and exposure to the Russian markets had a profound effect on economic activity and the banking sector. Five out of twelve banks, who held more than 40% of all deposits, experienced heavy distress.[24] These events revealed many flaws in the banking sector. As sources of the quake are cited lack of professional know-how, inadequate risk-management and risky portfolios, overexposure to foreign markets, insufficient capital adequacy, weak

[23]Adahl (2002), p.108.
[24]Chen et al. (2006), p.6

supervision and ill-practices.[25]

Between 1996 and 1998 the number of credit institutions went down from 15 to 6. The banks either consolidated, went insolvent or were acquired by large foreign institutions. Between the end of 1997 and the beginning of 2002, the central bank (Eesti Pank) took many actions to stabilize the sector. It introduced a minimum capital requirement, a maximum to the lending to a single borrower and put limits on the allowed foreign exchange exposure. Several laws such as the credit institutions law and the law on the Estonian Financial Supervision Agency (EFSA) were amended.[26] The latter came in effect in 2001, while EFSA itself started operations in 2002, which coincides perfectly with the predictions of the model that the risk premium was reduced significantly and almost vanished.

The financial crisis met a much stronger banking sector. The turmoil from 1997 saw the acquisition of many banks by larger foreign entities, primarily from Sweden which helped strengthen of the sector. No credit institutions went insolvent throughout the downturn. To face any potential runs, Eesti Pank secured an agreement with the Swedish Riskbank for liquidity support that was in place in March-December 2009.[27] As the currency board was in place, the Estonian central bank could not act as a lender of last resort and therefore turned to the parent banks for cooperation in case of liquidity shortage. This is in line with the model's estimation that higher risk was present in the period from 2008Q3 to 2009Q3. The spread between TALIBOR and EURIBOR existed longer but the specification allows for internal sources of the discrepancy — through the net foreign asset position channel.

4.3. Impulse Responses

The Markov-Switching specification has a considerable advantage over the baseline DSGE case. During stable times, the domestic rates do not deviate far from their counterparts and therefore any shocks to them should have little to no effect. On the other hand, under stressful times, like the Estonian banking crisis, the markets and the economy as a whole are much more sensitive to any disturbance in the banking sector. Hence, shocks would be more pronounced and their effects longer lasting. Figure 5 shows the impulse responses following a risk-premium shock under the standard model and the two regimes of the main Markov-Switching specification. As two volatilities can be estimated, the propagation effects of the size of the shocks can be then explored separately.

[25] See Adahl (2002).

[26] For more details and a timeline of all of the events see Adahl (2002).

[27] See Purfield and Rosenberg (2008), p.27.

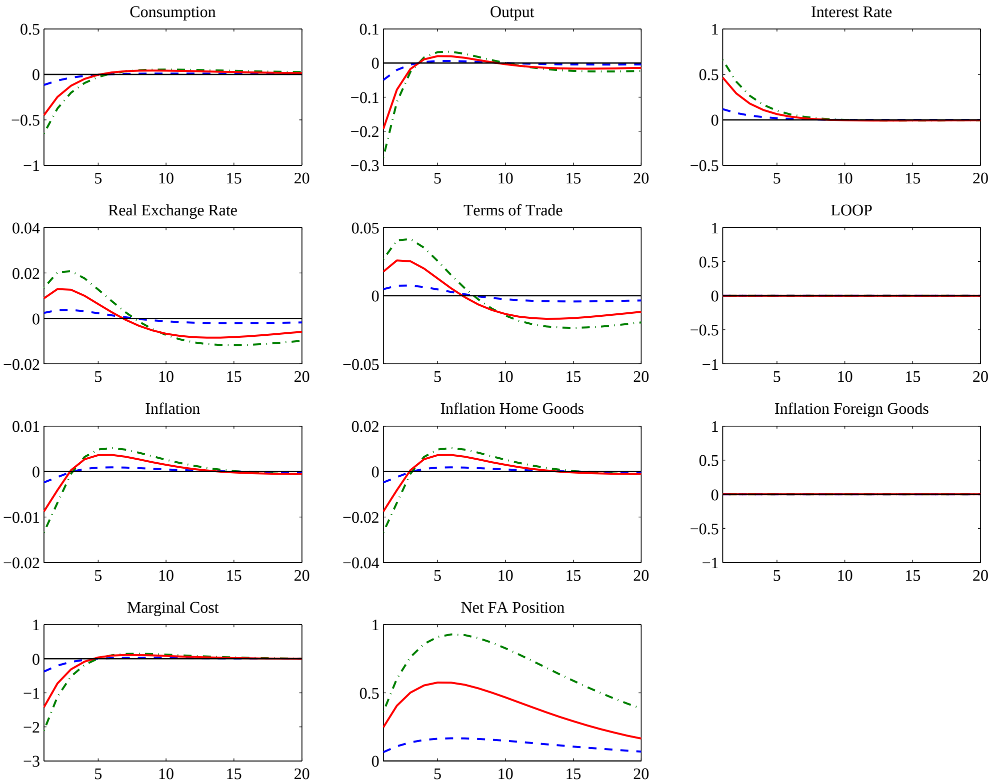


Figure 5: Impulse Responses following a risk premium shock for State 1: $\sigma_\phi(low)$ (- -), State 2: $\sigma_\phi(high)$ (-.-) and the no-switching version $\mathcal{M}_1$ (—)

The short run implications of the states of this model are quite distinct from each other and as suggested, the static DSGE specification produces a mix of both — overestimating the effects of shocks to the interest rate during "normal" periods and understating the impacts during crises. The reactions of the macroeconomic variables to a shock of the size from the low state are almost negligible (dashed blue line on the figure). When the currency board is stable, domestic agents have high confidence in the system, stemming in part from the macroeconomic conditions and from the reputation of the parent monetary authority. Output and consumption fall only slightly with a fast return to the steady state — in about two quarters. Inflation hardly reacts and so do the terms of trade. The strongest effect is in the marginal costs, which also stabilize in two to three quarters. $\mathcal{M}_1$ (in solid red) overestimates the fluctuations of the variables. Output and inflation responses overshoot by far the actual responses. The model also suggests worsening of the terms of trade and real exchange rate depreciation after 6 quarters.

In contrast, the responses in the high state (dash-dot green) are naturally much more pronounced than in the first regime. High risk-premium has long-lasting effects on the economy. Apart from consumption, almost no other variable returns to its steady state level within the first three years. A notable implication of the model is that stronger risk-premium shocks lead to cyclical behaviour of many macroeconomic variables. Sharper increase of the interest rate leads to a decrease in the price level only temporarily — for about three quarters — and is then followed by an increase in inflation. Thus in a currency board scenario, interest rate-related shocks during a crisis would not only lead to decrease in output but would also put inflationary pressure on the economy. The cyclical behaviour is also present in other variables such as the terms of trade, the real exchange rate and output to a certain extent. What needs to be taken into consideration here is that the actual duration of domestic price contracts is most probably shorter than estimated. With no change in foreign prices abroad and at home, the law of one price continues to hold and the only effect on inflation comes from the share of domestic goods in the consumer basket.

The long-run implications of the model may be explored through the means of variance decomposition. Under a regime switching system, the volatility is conditioned on the states.[28] Consumption is mainly driven by preferences, which is a standard result, especially in the absence of habit formation. About 70% of the variance in consumption in the long run is explained by the preference shocks. Interest rate shocks play almost no role in the variation in consumption in the long run under conditioning of the low state, but amount to almost 2% in the high state. Inflation is mainly driven by foreign price shocks (75%) and then domestic prices (17%), which is not a surprising result for Estonia, considering its trade background. Output is mostly driven by domestic price shocks and it seems that technology shocks do not play a significant role. However, the high coefficient of the price stickiness θ_H plays an important role here, as costs push shock would have a lasting effect and the number has an upward bias. It is interesting to note that during normal times, risk-premium shocks do not induce any volatility to output (0%), which suggest that the stable currency board can be a good monetary stabilizer in the long run.

Figure 6 shows the decomposition of the interest rate, for 12 periods ahead. The volatility of the interest rate is influenced mostly by risk premium shocks (63%) and foreign monetary policy shocks (30%), conditioning on the low state, but is dominated by the risk premium in the $high$ state. While in the former regime the effects of the two shocks diminish with time, in the latter state even after 40 periods risk premium shocks can explain up to 80% of the volatility in the interest rates. This illustrates the sensitivity and persistence of such shocks in the interest rate series. Hence, in the event of crisis it is

[28]The full tables for all variables can be found in the Appendix.

important that the authorities step in to calm the markets and reduce the risk premium.

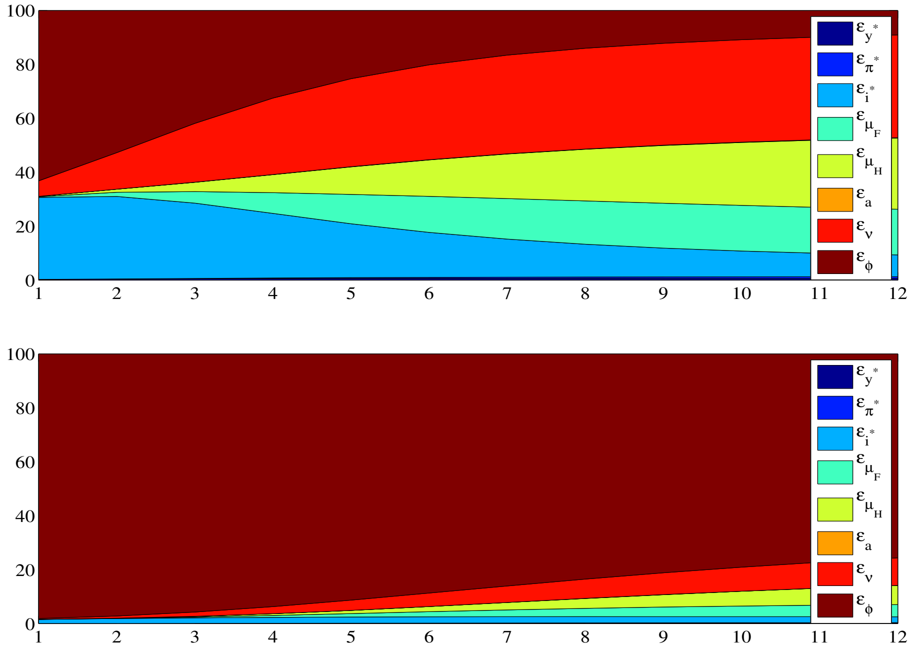


Figure 6: Variance decomposition of the interest rate for State 1: $\sigma_\phi(low)$ and State 2: $\sigma_\phi(high)$

The model is also able to match the volatility of the data. Table 3 displays the standard error of the actual variables and the moments based on 5000 simulations of the model from the posterior based on the sample length. Output has a standard deviation of 4.5%, while the model would generate 3.5%. Considering the model cannot explain the financial crisis and the huge dip in GDP, this looks like a good value. Consumption is matched well with an estimated error of 4.2% compared to the actual 4.05%. Inflation volatility is well matched and the interest rate is estimated to have more than twice the volatility in crises times.

Table 3: Standard deviation of the actual data and implied by the model based on 5000 random simulations

	y	c	π	i	y^*	π^*	i^*
data	4.5103	4.0453	0.8323	0.4635	1.3184	0.3754	0.2169
$\mathcal{M}_2$: State 1	3.4937	4.1860	0.8307	0.2480	1.2781	0.4434	0.1710
$\mathcal{M}_2$: State 2	3.5142	4.2666	0.8291	0.6966	1.2705	0.4443	0.1708

5. Robustness

This section evaluates three different Markov-Switching specifications to test the robustness of the main results. The first check — $\mathcal{M}_3$ — allows for regime shifts in the volatilities of all Estonian shocks to ensure that the switching coefficients in $\mathcal{M}_2$ do not pick up other peculiarities of the data. The next model — $\mathcal{M}_4$ — deals with the other coefficient in the interest rate equation — the debt elasticity χ. It is allowed to switch along with the volatility of the exogenous component. Finally, $\mathcal{M}_5$ follows a common path in the literature and does not use any detrending on the interest rates. Table 4 provides a summary.[29]

Table 4: Differences between the specifications

$\mathcal{M}_1$:	No regime shifts.
$\mathcal{M}_2$:	Switching in the volatility of the risk premium σ^2_ϕ.
$\mathcal{M}_3$:	Switching in the volatility in other structural shocks: σ^2_a, σ^2_ϑ, $\sigma^2_{\mu_H}$, $\sigma^2_{\mu_F}$, σ^2_ϕ.
$\mathcal{M}_4$:	Switching in σ^2_ϕ and χ.
$\mathcal{M}_5$:	Switching in σ^2_ϕ where the interest rate data is not detrended.

5.1. $\mathcal{M}_3$: Simultaneous switching in other shocks

In a general equilibrium model, the flexibility of a Markov-Switching framework holds a caveat — peculiarities of one time series my propagate through several variables and end up in the extra parameters. Time-variation is a standard feature in macroeconomic and financial data and it might be that the extra parameter acts as a "pressure valve" to the model. If that is the case, allowing more parameters to switch simultaneously would distort the estimates of the risk-premium volatility. Hence, in this exercise, heteroskedasticity for all of the Estonian shocks is allowed: $\sigma^2_a(s_t)$, $\sigma^2_\vartheta(s_t)$, $\sigma^2_{\mu_H}(s_t)$, $\sigma^2_{\mu_F}(s_t)$, $\sigma^2_\phi(s_t)$. The priors of the parameters are left identical in both states. Table 5 displays the coefficients for all models. Columns five to seven represent $\mathcal{M}_2$ and $\mathcal{M}_3$.

Overall the estimates are extremely similar to the original specification and few are worth noting. The stochastic volatility of the risk premium in the third model is $\sigma^2_\phi(l) = 0.121$ and $\sigma^2_\phi(h) = 0.673$ at the mean which is almost identical to the reported values for $\mathcal{M}_2$ and the estimated distributions are mostly

[29] Two further robustness checks have been carried out — a linear detrending and a model with a VAR representation. The results are similar across all specifications. The tables for the two additional models, as well as convergence diagnostics, trace and distribution plots, and recursive means plots for all robustness models are available upon request.

overlapping. This shows that the main results are a feature of the interest-rate time series alone. From the non-switching parameters, only the inverse of the elasticity of substitution is higher ($\frac{1}{\sigma} = \frac{1}{2.1}$ compared to $\frac{1}{2.4}$ in $\mathcal{M}_2$) at the mean and the distribution is slightly shifted to the left. This is complemented by a smaller volatility of the preference shock (in both states) as higher willingness to substitute goods across time means less impact of shocks on consumption. The marginal density of this model is $\mathbb{M}_3 = -409.2437$ which is worse than the reported value $\mathbb{M}_2 = -405.5175$ in the original specification.

5.2. $\mathcal{M}_4$: Switching in both the volatility and debt elasticity

The interest rate equation (12) pins down the spread between the two rates as a composite component. Allowing for time variation in the exogenous part then explains part of the spread during crisis as a result of external factors. Europe is currently experiencing severe difficulties with public debt levels and worsening debt positions are being watched closely. Estonia's public debt has been kept small (about 6% to 7% of GDP), however, before and during the financial crisis, Estonia's net external debt fell to almost -40% of GDP before returning to zero levels in 2012. It might be that the sensitivity towards indebtedness has simply increased, which would distort the results. In the following, a model with both switching the debt elasticity parameter χ and in the volatility of risk-premium is estimated.

$$i_t = i_t^* - \chi(s_t)d_t - \phi_t$$

$$\phi_t = \rho_\phi \phi_{t-1} + \varepsilon_t^\phi \quad \text{with} \quad \varepsilon_t^\phi \sim N(0, \sigma_\phi^2(s_t))$$

The third and fourth-to-last columns of table 4 present the parameters at the mean with 95% probability intervals for $\mathcal{M}_4$. Again, most non-switching parameters are equal between this and the original MS-model up to a second or third digit after the decimal. The risk premium volatility in the *low* state $\sigma_\phi(low) = 0.119$ is identical for both, while the *high* state parameter has closely overlapping distributions with $\sigma_\phi(high) = 0.665$ and $\sigma_\phi(high) = 0.680$ for $\mathcal{M}_2$ and $\mathcal{M}_4$ respectively. This suggest that the estimates are robust to this specification. The other important parameter in the model χ takes the values $\chi(low) = 0.014$ and $\chi(high) = 0.024$ at the mean. The probability intervals, however, reveal that the distribution of $\chi(high)$ includes all the possible values for $\chi(low)$ in 95% of the cases. The intervals are [0.006, 0.025] for the *low* state and [0.005, 0.051] for the *high* state. A mean value of $\chi(high) = 0.024$ implies that in almost 50% of the cases in the *high* state, the value of $\chi(high)$ could be obtained as if $\chi = \chi(low)$. There is no conclusive evidence regarding the existence of two significantly different values. The marginal density of $\mathcal{M}_4$ is $\mathbb{M}_4 = -405.0004$ which is close to the original $\mathbb{M}_2 = -405.5175$, yet slightly better.

Table 5: Alternative specifications and the benchmark model $\mathcal{M}_2$

	Distribution	Prior Mean	$\mathcal{M}_1$	$\mathcal{M}_2 : S_t = 1$	$\mathcal{M}_2 : S_t = 2$	$\mathcal{M}_3 : S_t = 1$	$\mathcal{M}_3 : S_t = 2$	$\mathcal{M}_4 : S_t = 1$	$\mathcal{M}_4 : S_t = 2$	$\mathcal{M}_5 : S_t = 1$	$\mathcal{M}_5 : S_t = 2$
p_{11}	$Beta$	0.900	—	0.936 [0.862, 0.984]	—	0.935 [0.848, 0.989]	—	0.929 [0.851, 0.982]	—	0.958 [0.906, 0.991]	—
p_{22}	$Beta$	0.900	—	0.942 [0.852, 0.993]	—	0.937 [0.863, 0.985]	—	0.942 [0.850, 0.993]	—	0.932 [0.858, 0.981]	—
β	PM	0.995	0.995	0.995	—	0.995	—	0.995	—	0.995	—
φ	$Gamma$	2.000	1.985 [1.608, 2.404]	1.982 [1.598, 2.399]	—	1.992 [1.599, 2.422]	—	1.982 [1.601, 2.395]	—	1.990 [1.606, 2.398]	—
θ_H	$Beta$	0.750	0.910 [0.880, 0.938]	0.912 [0.882, 0.939]	—	0.910 [0.879, 0.938]	—	0.913 [0.883, 0.940]	—	0.938 [0.918, 0.955]	—
θ_F	$Beta$	0.500	0.631 [0.544, 0.717]	0.645 [0.556, 0.733]	—	0.655 [0.566, 0.745]	—	0.644 [0.557, 0.730]	—	0.723 [0.642, 0.801]	—
α	PM	0.500	0.500	0.500	—	0.500	—	0.500	—	0.500	—
σ	$Gamma$	1.000	2.339 [1.371, 3.694]	2.424 [1.434, 3.800]	—	2.130 [1.262, 3.377]	—	2.438 [1.454, 3.808]	—	6.750 [4.789, 9.192]	—
η	$Gamma$	2.000	2.366 [2.011, 2.760]	2.411 [2.062, 2.781]	—	2.421 [2.068, 2.816]	—	2.405 [2.046, 2.793]	—	2.259 [1.924, 2.624]	—
δ_H	$Beta$	0.500	0.215 [0.094, 0.371]	0.217 [0.096, 0.369]	—	0.221 [0.100, 0.372]	—	0.216 [0.095, 0.369]	—	0.229 [0.101, 0.385]	—
δ_F	$Beta$	0.500	0.590 [0.386, 0.786]	0.594 [0.395, 0.788]	—	0.539 [0.333, 0.745]	—	0.590 [0.387, 0.788]	—	0.577 [0.372, 0.772]	—
χ	$Gamma$	0.010	0.028 [0.014, 0.043]	0.017 [0.006, 0.029]	—	0.015 [0.006, 0.028]	—	0.014 [0.006, 0.025]	0.024 [0.005, 0.051]	0.025 [0.021, 0.030]	—
ρ_a	$Beta$	0.700	0.698 [0.520, 0.851]	0.703 [0.526, 0.854]	—	0.701 [0.525, 0.854]	—	0.699 [0.524, 0.850]	—	0.700 [0.526, 0.852]	—
ρ_{μ_F}	$Beta$	0.700	0.700 [0.531, 0.847]	0.708 [0.537, 0.853]	—	0.714 [0.555, 0.852]	—	0.706 [0.536, 0.852]	—	0.715 [0.547, 0.858]	—
ρ_{μ_H}	$Beta$	0.700	0.650 [0.480, 0.807]	0.670 [0.499, 0.821]	—	0.660 [0.491, 0.814]	—	0.659 [0.488, 0.816]	—	0.677 [0.509, 0.825]	—
ρ_ν	$Beta$	0.700	0.697 [0.531, 0.842]	0.695 [0.523, 0.841]	—	0.681 [0.507, 0.828]	—	0.693 [0.523, 0.838]	—	0.694 [0.521, 0.840]	—
ρ_ϕ	$Beta$	0.700	0.640 [0.474, 0.789]	0.646 [0.487, 0.792]	—	0.638 [0.469, 0.790]	—	0.641 [0.476, 0.789]	—	0.634 [0.469, 0.784]	—
c_{y^*}	$Beta$	0.850	0.884 [0.790, 0.968]	0.883 [0.786, 0.969]	—	0.885 [0.787, 0.970]	—	0.885 [0.792, 0.968]	—	0.878 [0.782, 0.963]	—
c_{π^*}	$Beta$	0.850	0.545 [0.379, 0.722]	0.548 [0.382, 0.722]	—	0.549 [0.382, 0.724]	—	0.551 [0.386, 0.726]	—	0.553 [0.381, 0.740]	—
c_{i^*}	$Beta$	0.850	0.861 [0.783, 0.933]	0.857 [0.782, 0.925]	—	0.851 [0.775, 0.920]	—	0.857 [0.776, 0.928]	—	0.953 [0.928, 0.976]	—
σ_{μ_F}	$IGamma$	1.000	1.225 [0.857, 1.673]	1.159 [0.799, 1.618]	—	0.966 [0.611, 1.442]	1.245 [0.810, 1.836]	1.160 [0.798, 1.611]	—	0.983 [0.664, 1.379]	—
σ_{μ_H}	$IGamma$	1.000	0.458 [0.326, 0.620]	0.425 [0.298, 0.586]	—	0.403 [0.275, 0.570]	0.474 [0.327, 0.662]	0.433 [0.312, 0.583]	—	0.405 [0.290, 0.548]	—
σ_a	$IGamma$	1.000	0.855 [0.209, 2.358]	1.060 [0.205, 3.474]	—	0.769 [0.207, 2.007]	1.040 [0.212, 3.117]	0.816 [0.206, 2.346]	—	1.088 [0.212, 3.966]	—
σ_ν	$IGamma$	1.000	11.265 [7.040, 17.150]	11.438 [7.154, 17.458]	—	9.569 [5.430, 15.636]	10.440 [6.404, 16.143]	11.473 [7.203, 17.380]	—	33.342 [24.976, 44.307]	—
σ_ϕ	$IGamma$	0.800	0.472 [0.406, 0.548]	0.119 [0.090, 0.156]	0.665 [0.533, 0.831]	0.121 [0.092, 0.160]	0.673 [0.535, 0.842]	0.119 [0.090, 0.156]	0.680 [0.537, 0.866]	0.065 [0.971, 1.610]	1.249 [0.049, 0.081]
σ_{y^*}	$IGamma$	1.000	0.684 [0.592, 0.791]	0.685 [0.593, 0.795]	—	0.686 [0.593, 0.798]	—	0.684 [0.591, 0.793]	—	0.685 [0.588, 0.798]	—
σ_{π^*}	$IGamma$	1.000	0.375 [0.321, 0.438]	0.376 [0.321, 0.440]	—	0.376 [0.322, 0.440]	—	0.376 [0.321, 0.440]	—	0.377 [0.323, 0.443]	—
σ_{i^*}	$IGamma$	1.000	0.100 [0.086, 0.116]	0.100 [0.086, 0.116]	—	0.100 [0.086, 0.118]	—	0.100 [0.086, 0.117]	—	0.104 [0.089, 0.121]	—
$\mathbb{M}$:			-430.723	-405.5175		-409.2437		-405.0004		-415.8807	

5.3. $\mathcal{M}_5$: No detrending of the interest rate

In this specification the interest rate series are taken "as is" and the last two columns of table 4 contain the estimated coefficients. Due to lack of detrending, the spread between the series is larger. This is reflected in the estimated coefficients — $\sigma(\text{"}low\text{"}) = 0.05$ and $\sigma(\text{"}high\text{"}) = 1.2$ — an even stronger result. Figure 7 plots the probabilities of the two states and does not differ significantly from the main model. There is the zero probability assigned to the period before the first crisis, whereas in the main MS-model, there was positive but inconclusive probability attached to that period. In a way, this highlights the benefits of a structural general equilibrium model, where the interaction between the variables is sophisticated: the spread of the interest rates between 1996 and 1997Q2 is as big as the periods of mounting pressure before the financial crisis — 2007Q2 to 2008Q2, yet the model is able to identify that the former is not caused by a higher risk premium. Also, the main findings are robust — the 1997–1999 bank crisis and the global financial turmoil up to the third quarter of 2009 have been characterized with higher risk-premium.

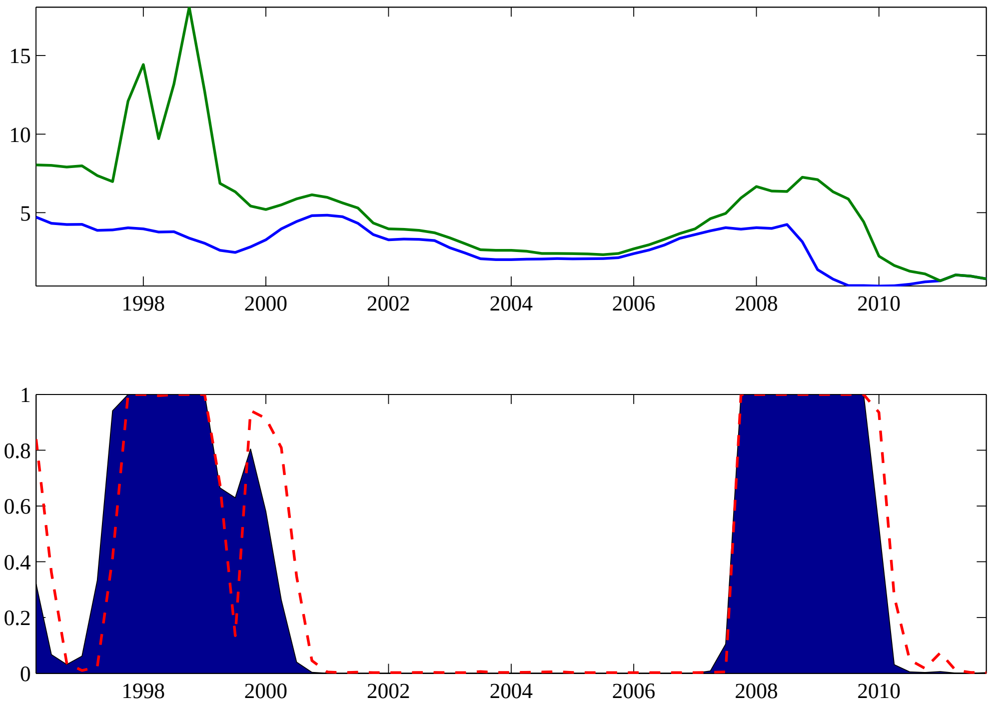


Figure 7: Top: Interbank interest rates in %, Estonia's TALIBOR (—) and EURIBOR (- -)
Note: Bottom: Smoothed (⊓) and not-smoothed (- -) probability of a high state.

With regards to the rest of the parameters, the only major difference is in the coefficient of risk aversion, which is much higher. This is in line with the higher standard deviation, as more risk into the system, especially associated with the means of inter-temporal substitution, would increase the risk aversion of the agents. In terms of marginal density, the model is not preferred over the other models with detrended interest rate. It ranks as second worst with -415.88.

6. Conclusions

In the baseline DSGE model of an economy with a currency board, foreign and domestic interest rates are typically modelled as an identity. In reality, the rates converge, yet in times of crises a persistent positive spread usually opens. When risk-premium shock are studied, the responses of the variables would not be representing reality properly. If the currency board is stable, innovations to the interest rates would not be expected to have a profound effect on the economy. In stressful times, however, the system is much more sensitive to disturbances in the economy.

This paper tackles these issues by developing a model where the interest rate is a function of several endogenous and exogenous variables utilizing a Markov-Switching framework to capture the periods of persistent spread and increased sensitivity of the variables. The domestic interest rate is derived as a function of the foreign rate and a risk premium. The latter is composed of two parts — an exogenous component and an endogenous function that captures the indebtedness of the country. The volatility of the exogenous part is then allowed to switch among two regimes to capture any change in the premium. The model is applied to Estonia, which is very suitable for the exercise, having experienced a banking crisis, a financial crisis and booming periods in-between. The stressful periods are well identified as the time-variation in the volatility of the risk-premium is captured from the data. The impulse responses show that the static DSGE model would underperform the switching version as it tends to average the volatility of the series. Stronger shocks produce a cyclical behaviour in many series, most notably inflation, where a sudden sharp increase in the interest rates lead only temporary to deflation and then to inflation. In the long run, the stable currency board minimizes the effects of risk-premium shocks — in Estonia, during the booming periods these shocks did not contribute to the volatility of output at all.

References

ADAHL, M. (2002): The Development of the Banking Systems in Estonia, Latvia and Lithuania Since Independence — The Internationalization of Baltic Banking (1998-2002).

AN, S. AND SCHORFHEIDE, F. (2007): Bayesian Analysis of DSGE Models. *Econometric Reviews*, No. 26(2), pp. 113-172.

ANDREASEN, M. (2008): How to Maximize the Likelihood Function for a DSGE Model. CREATES Research Papers 2008-32, School of Economics and Management, University of Aarhus.

BENIGNO, P. (2001): Price Stability with Imperfect Financial Integration. Working Paper DP2584, CERP.

BIANCHI, F. (2012): Regime switches, Agents' Beliefs, and Post-World War II U.S. Macroeconomic Dynamics. Mpra paper, (forthcoming in *Review of Economic Studies*).

CALVO, G. (1983): Staggered Prices in a Utility Maximizing Framework. *Journal of Monetary Economics*, No. 12. pp. 383-398.

CHEN, X. AND MACDONALD, R. (2012): Realized and Optimal Monetary Policy Rules in an Estimated Markov-Switching DSGE Model of the United Kingdom. *Journal of Money, Credit and Banking*, No. 44(6), pp. 1091-1116.

CHEN, Y.-F., FUNKE, M., AND MÄNNASOO, K. (2006): Extracting Leading Indicators of Bank Fragility from Market Prices Estonia Focus.

CHO, S. (2011): Characterizing Markov-Switching Rational Expectations Models. Working paper.

DAVIG, T. AND LEEPER, E. M. (2007): Fluctuating Macro Policies and the Fiscal Theory. In NBER Macroeconomics Annual 2006, Volume 21, NBER Chapters, pp. 247-316. National Bureau of Economic Research, Inc.

FARMER, R., WAGGONER, D. F., AND ZHA, T. (2009): Understanding Markov-switching rational expectations models. *Journal of Economic Theory*, Vol. 144(5), pp. 1849-1867.

FARMER, R., WAGGONER, D. F., AND ZHA, T. (2010). Minimal State Variable Solutions To Markov-Switching Rational Expectations Models. Emory Economics 1003, Department of Economics, Emory University (Atlanta).

GALI, J. AND MONACELLI, T. (2002): Monetary Policy and Exchange Rate Volatility in a Small Open Economy. Working Paper 8905, NBER.

GALI, J. AND MONACELLI, T. (2005): Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *The Review of Economic Studies*, Vol. 72(3), pp. 707-734.

GELAIN, P. AND KULIKOV, D. (2009): An Estimated Dynamic Stochastic General Equilibrium Model for Estonia. Bank of Estonia Working Papers, 2009-5, Bank of Estonia, Tallinn.

HAMILTON, J. D. (1989): A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, Vol. 57(2), pp. 35-384.

HANSEN, N. (2006): The CMA Evolution Strategy: a Comparing Review. In Lozano, J. A., Larranaga, P., Inza, I., and Bengoetxea, E., editors, Towards a new evolutionary computation. Advances on estimation of distribution algorithms, pp. 75-102. Springer.

JUSTINIANO, A. AND PRESTON, B. (2010): Monetary policy and uncertainty in an empirical small open-economy model. *Journal of Applied Econometrics*, Vol. 25(1), pp. 93-128.

JUSTINIANO, A. AND PRIMICERI, G. E. (2008): The Time-Varying Volatility of Macroeconomic Fluctuations. *American Economic Review*, Vol. 98(3), pp. 604-641.

KIM, C.-J. AND NELSON, C. (1999): State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications. The MIT Press.

KOLLMANN, R. (2002): Monetary Policy Rules in the Open Economy: Effects on Welfare and Business cycles. *Journal of Monetary Economics*, Vol. 49, pp. 989-1015.

LIU, P. AND MUMTAZ, H. (2011): Evolving Macroeconomic Dynamics in a Small Open Economy: An Estimated Markov Switching DSGE Model for the UK. *Journal of Money, Credit and Banking*, Vol. 43(7), pp. 1443-1474.

LUBIK, T. AND SCHORFHEIDE, F. (2007): Do Central Banks Respond to Exchange Rate Movements? A Atructural Investigation. *Journal of Monetary Economics*, Vol. 54(4), pp. 1069-1087.

MCCALLUM, B. T. (1983): On non-uniqueness in rational expectations models : An attempt at perspective. *Journal of Monetary Economics*, Vol. 11(2), pp. 139-168.

MONACELLI, T. (2005): Monetary Policy in a Low Pass-Through. *Journal of Money, Credit and Banking*, Vol. 37(6), pp. 1047-1066.

PURFIELD, C. AND ROSENBERG, C. B. (2008): Adjustment under a Currency Peg : Estonia , Latvia and Lithuania during the Global Financial Crisis 2008-09. IMF working paper wp/10/213, IMF.

RANDVEER, M. AND DABUŠSINSKAS, A. (2006): Comparison of Pricing Behavior of Firms in the Euro Area and Estonia. Eesti Pank Working Paper Series No.8, Eesti Pank.

SARGENT, T., WILLIAMS, N., AND ZHA, T. (2009): The Conquest of South American Inflation. *Journal of Political Economy*, Vol. 117(2), pp. 211-256.

SCHMITT-GROHE, S. AND URIBE, M. (2003): Closing Small Open Economy Models. *Journal of International Economics*, Vol. 61(1), pp. 163-185.

SCHWAB, K. (2011): Global Competitiveness Report. Technical report, World Economic Forum.

SMETS, F. AND WOUTERS, R. (2003): An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association*, No. 1(5), pp. 1123-1175.

A Appendix

A1. Log-linearized system of equations

Endogenous Equations:

Euler equation:

$$c_t = E_t\{c_{t+1}\} + \frac{1}{\sigma}(E_t\{\pi_{t+1}\} - i_t) + \frac{1}{\sigma}(1 - \rho_\vartheta)\vartheta_t \tag{A.1}$$

Domestic Price Inflation:

$$(1 + \beta\delta_H)\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \delta_H \pi_{H,t-1} + \lambda_H mc_t + \mu_{H,t} \tag{A.2}$$

Import Price Inflation:

$$(1 + \beta\delta_F)\pi_{F,t} = \beta E_t\{\pi_{F,t+1}\} + \delta_F \pi_{F,t-1} + \lambda_F \psi_t + \mu_{F,t} \tag{A.3}$$

Market Clearing:

$$y_t - (1 - \alpha)c_t - \alpha\eta(s_t + q_t) = \alpha y_t^* \tag{A.4}$$

Law of One Price:

$$\psi_t = q_t - (1 - \alpha)s_t \tag{A.5}$$

Terms of Trade:

$$\triangle s_t = \pi_{F,t} - \pi_{H,t} \tag{A.6}$$

Nominal Exchange Rate:

$$\triangle e_t = 0 \tag{A.7}$$

Interest Rate Parity:

$$\pi_t^* - \pi_t = \triangle q_t \tag{A.8}$$

Marginal Cost:

$$mc_t = \sigma c_t + \varphi y_t + \alpha s_t - (1 + \varphi)a_t \tag{A.9}$$

CPI:

$$\pi = (1 - \alpha)\pi_{H,t} + \alpha\pi_{F,t} \tag{A.10}$$

Foreign Asset Budget Constraint:

$$c_t + d_t + \alpha(q_t + \alpha s_t) - \frac{1}{\beta} d_{t-1} = y_t \tag{A.11}$$

Interest Rate Reaction Function:

$$i_t = i_t^* - \chi d_t - \phi_t \tag{A.12}$$

Exogenous processes:

Domestic Shocks:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \quad \text{with} \quad \varepsilon_t^a \sim N(0, \sigma_a^2) \tag{A.13}$$
$$\vartheta_t = \rho_\vartheta \vartheta_{t-1} + \varepsilon_t^\vartheta \quad \text{with} \quad \varepsilon_t^\vartheta \sim N(0, \sigma_\vartheta^2) \tag{A.14}$$
$$\mu_{H,t} = \rho_\mu \mu_{H,t-1} + \varepsilon_t^{\mu_H} \quad \text{with} \quad \varepsilon_t^{\mu_H} \sim N(0, \sigma_{\mu_H}^2) \tag{A.15}$$
$$\mu_{F,t} = \rho_\mu \mu_{F,t-1} + \varepsilon_t^{\mu_F} \quad \text{with} \quad \varepsilon_t^{\mu_F} \sim N(0, \sigma_{\mu_F}^2) \tag{A.16}$$
$$\phi_t = \rho_\phi \phi_{t-1} + \varepsilon_t^\phi \quad \text{with} \quad \varepsilon_t^\phi \sim N(0, \sigma_\phi^2) \tag{A.17}$$

World Variables:

$$y_t^* = c_{y^*} y_{t-1}^* + \varepsilon_t^{y^*} \quad \text{with} \quad \varepsilon_t^{y^*} \sim N(0, \sigma_{y^*}^2) \tag{A.18}$$
$$\pi_t^* = c_{\pi *} \pi_{t-1}^* + \varepsilon_t^{\pi^*} \quad \text{with} \quad \varepsilon_t^{\pi^*} \sim N(0, \sigma_{\pi^*}^2) \tag{A.19}$$
$$i_t^* = c_{i *} i_{t-1}^* + \varepsilon_t^{i^*} \quad \text{with} \quad \varepsilon_t^{i^*} \sim N(0, \sigma_{i^*}^2) \tag{A.20}$$

A2. Solving a MSRE Model

This section will sketch the solution method employed in the paper. For details and proofs, see Cho (2011). The model is cast in the following state-space form:

$$X_t = E_t\{A(s_t, s_{t+1}) X_{t+1}\} + B(s_t) X_{t-1} + C(s_t) Z_t \tag{A.21}$$

with Z_t following an AR(1) process.[30] From the perspective of time period t by forward iteration the model in period $t+k$ may be represented by:

$$X_t = E_t\{M_k(s_t, s_{t+1}, ..., s_{t+k}) X_{t+k}\} + \Omega_k(s_t) X_{t-1} + \Gamma_k(s_t) Z_t \tag{A.22}$$

where $\Omega_1(s_t) = B(s_t)$, $\Gamma_1(s_t) = C(s_t)$ and for $k = 2, 3...$

$$\begin{aligned}
\Omega_k(s_t) &= \Xi_{k-1}(s_t)^{-1} B(s_t) \\
\Gamma_k(s_t) &= \Xi_{k-1}(s_t)^{-1} C(s_t) + E_t\{F_{k-1}(s_t, s_{t+1}) \Gamma_{k-1}(s_{t+1})\} R \\
\Xi_{k-1}(s_t) &= (I_n - E_t\{A(s_t, s_{t+1}) \Omega_{k-1(s_{t+1})}\}) \\
F_{k-1}(s_t, s_{t+1}) &= \Xi_{k-1}(s_t)^{-1} A(s_t, s_{t+1})
\end{aligned}$$

[30]Note that $A(s_t, s_{t+1}) = B1(s_t)^{-1} A1(s_t, s_{t+1})$ in (23). B and C are similarly defined.

It may be shown that given initial values, under some regularity conditions such as invertibility of Ξ $\forall k$ the sequence $E_t\{M_k(s_t, s_{t+1}, ...s_{t+k})X_{t+k}\}$ is well defined, unique and real-valued. Taking $\lim\limits_{k\to\infty}$, the model in (A.22) is said to be Forward Convergent if the parameter matrices are convergent, i.e: $\lim\limits_{k\to\infty} \Omega_k(s_t) = \Omega^*(s_t)$; $\lim\limits_{k\to\infty} \Gamma_k(s_t) = \Gamma^*(s_t)$ and $\lim\limits_{k\to\infty} F_k(s_t, s_{t+1}) = F^*(s_t, s_{t+1})$. If

$$\lim_{k\to\infty} E_t\{M_k(s_t, s_{t+1}, ..., s_{t+k})X_{t+k}\} = 0_{n\times 1} \tag{A.23}$$

then the solution is:

$$X_t = \Omega^*(s_t)X_{t-1} + \Gamma^*(s_t)Z_t \tag{A.24}$$

Equation (A.23) is called the *no-bubble condition* and is a solution property of the model. As k tends to infinity, this condition should hold and all solutions where it does not should be ruled out as they are not economically relevant. Thus, if the model is forward convergent and (A.23) is satisfied, then (A.24) is the only relevant MSV solution to the model cast in the form of (23).

The existence of (A.24) alone is necessary but not sufficient condition for determinacy, due to the volatility induced by the regime-switching feature. The MSV solution is only the *fundamental part* of the solution, but there may exist a non-fundamental part that is arbitrary and there may be a multiplicity of equilibria. Assuming the non-fundamental component takes the form

$$W_t = E_t\{F(s_t, s_{t+1})W_{t+1}\} \tag{A.25}$$

then the concept for determinacy and indeterminacy deals with interaction of the matrices when switching between states. Defining

$$\Psi_{\Omega^*\times\Omega^*} = [p_{ij}\Omega_j^* \otimes \Omega_j^*] \qquad and \qquad \Psi_{F^*\times F^*} = [p_{ij}F_j^* \otimes F_j^*]$$

then, mean-square stability is characterized by

$$r_\sigma(\Psi_{\Omega^*\times\Omega^*}) < 1 \qquad\qquad r_\sigma(\Psi_{F^*\times F^*}) \leq 1 \tag{A.26}$$

The intuition behind these conditions is straightforward. The first concerns the transition between the matrices $\Omega^*(s_t)$ of the fundamental part of the solution (A.24). As long as the highest of the eigenvalues is smaller than one, the system would be stable under regime-switching. The F matrix governs non-fundamental switching part and as long the highest eigenvalue lies on or within the unit-circle, the forward solution is the determinate equilibrium.

A3. $\mathcal{M}_1$: Convergence diagnostics: figures and tables

Table 6: Left: Autocorrelation among the draws, based on a sample of 10000. Right: Raferty-Lewis convergence diagnostics with q=0.025, r=0.1, s=0.95. I-stat should be less than 5.

	Lag 1.	Lag 5	Lag 10	Lag 50
φ	0.615	0.104	0.008	0.009
θ_H	0.658	0.170	0.054	0.005
θ_F	0.630	0.095	0.035	0.010
σ	0.737	0.265	0.046	0.038
η	0.636	0.141	0.015	–0.002
δ_H	0.655	0.160	0.056	–0.008
δ_F	0.626	0.105	0.026	0.030
χ	0.617	0.084	–0.025	0.025
ρ_a	0.617	0.089	0.011	–0.011
ρ_{μ_F}	0.627	0.133	0.023	–0.005
ρ_{μ_H}	0.633	0.108	–0.003	–0.021
ρ_ν	0.632	0.131	0.034	0.010
ρ_ϕ	0.615	0.088	0.013	–0.012
c_{y^*}	0.631	0.126	0.013	0.002
c_{π^*}	0.643	0.106	–0.008	–0.016
c_{i^*}	0.636	0.114	0.007	–0.005
σ_{μ_F}	0.661	0.143	0.044	0.014
σ_{μ_H}	0.683	0.186	0.049	0.010
σ_a	0.924	0.740	0.604	0.126
σ_ν	0.737	0.274	0.058	0.048
σ_ϕ	0.643	0.130	0.016	–0.019
σ_{y^*}	0.626	0.114	0.028	0.005
σ_{π^*}	0.653	0.126	0.007	–0.003
σ_{i^*}	0.644	0.088	0.015	–0.020

	Thin	Burn	Total(N)	(Nmin)	I-stat
φ	1.000	5.000	1510.000	937.000	1.612
θ_H	1.000	5.000	1510.000	937.000	1.612
θ_F	1.000	5.000	1510.000	937.000	1.612
σ	1.000	5.000	1510.000	937.000	1.612
η	1.000	5.000	1510.000	937.000	1.612
δ_H	1.000	5.000	1510.000	937.000	1.612
δ_F	1.000	5.000	1510.000	937.000	1.612
χ	1.000	5.000	1510.000	937.000	1.612
ρ_a	1.000	5.000	1510.000	937.000	1.612
ρ_{μ_F}	1.000	5.000	1510.000	937.000	1.612
ρ_{μ_H}	1.000	5.000	1510.000	937.000	1.612
ρ_ν	1.000	5.000	1510.000	937.000	1.612
ρ_ϕ	1.000	5.000	1510.000	937.000	1.612
c_{y^*}	1.000	5.000	1510.000	937.000	1.612
c_{π^*}	1.000	5.000	1510.000	937.000	1.612
c_{i^*}	1.000	5.000	1510.000	937.000	1.612
σ_{μ_F}	1.000	5.000	1510.000	937.000	1.612
σ_{μ_H}	1.000	5.000	1510.000	937.000	1.612
σ_a	1.000	5.000	1510.000	937.000	1.612
σ_ν	1.000	5.000	1510.000	937.000	1.612
σ_ϕ	1.000	5.000	1510.000	937.000	1.612
σ_{y^*}	1.000	5.000	1510.000	937.000	1.612
σ_{π^*}	1.000	5.000	1510.000	937.000	1.612
σ_{i^*}	1.000	5.000	1510.000	937.000	1.612

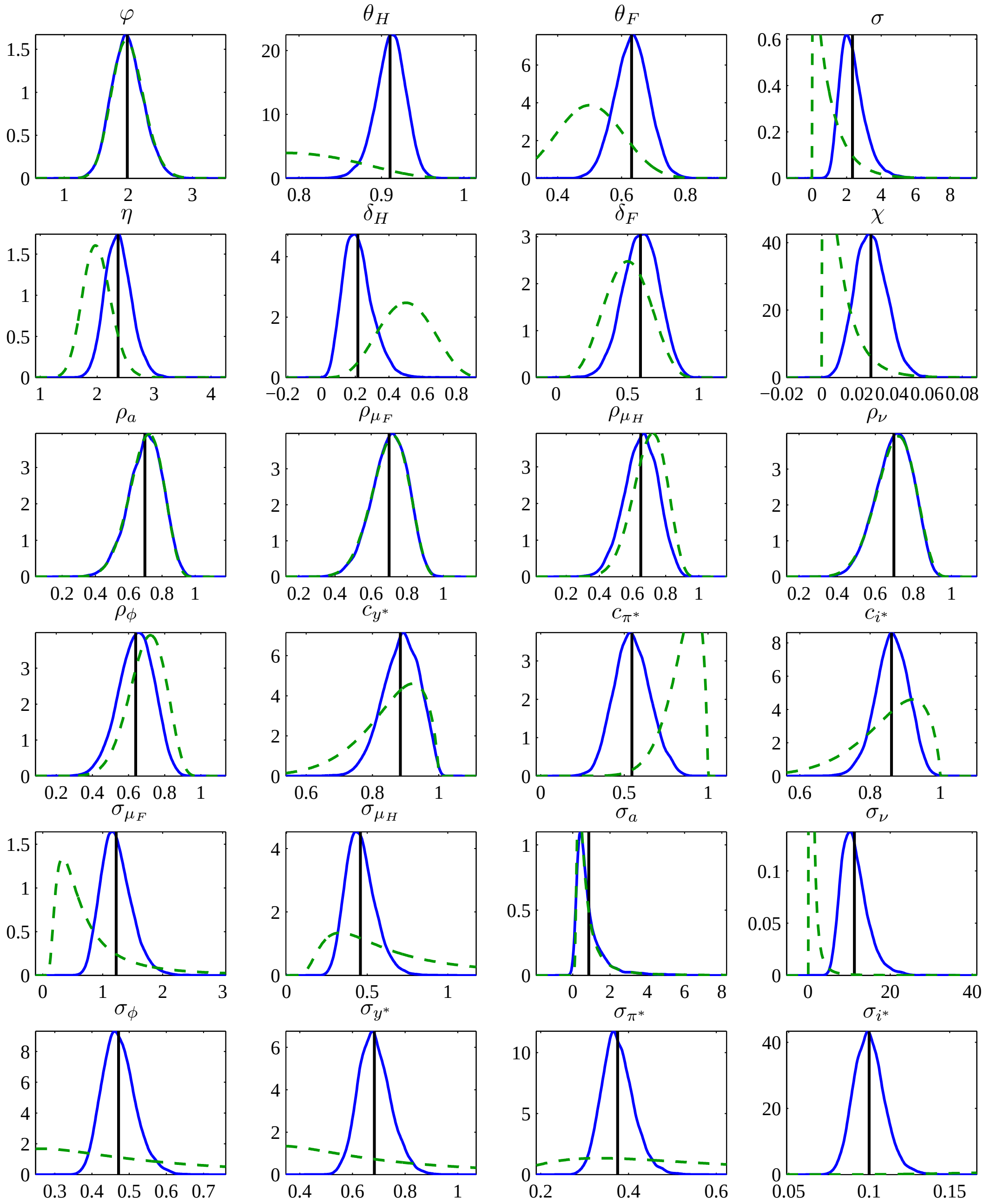


Figure 8: Prior (dashed) and posterior (solid) distributions of $\mathcal{M}_1$ (no Markov-Switching)

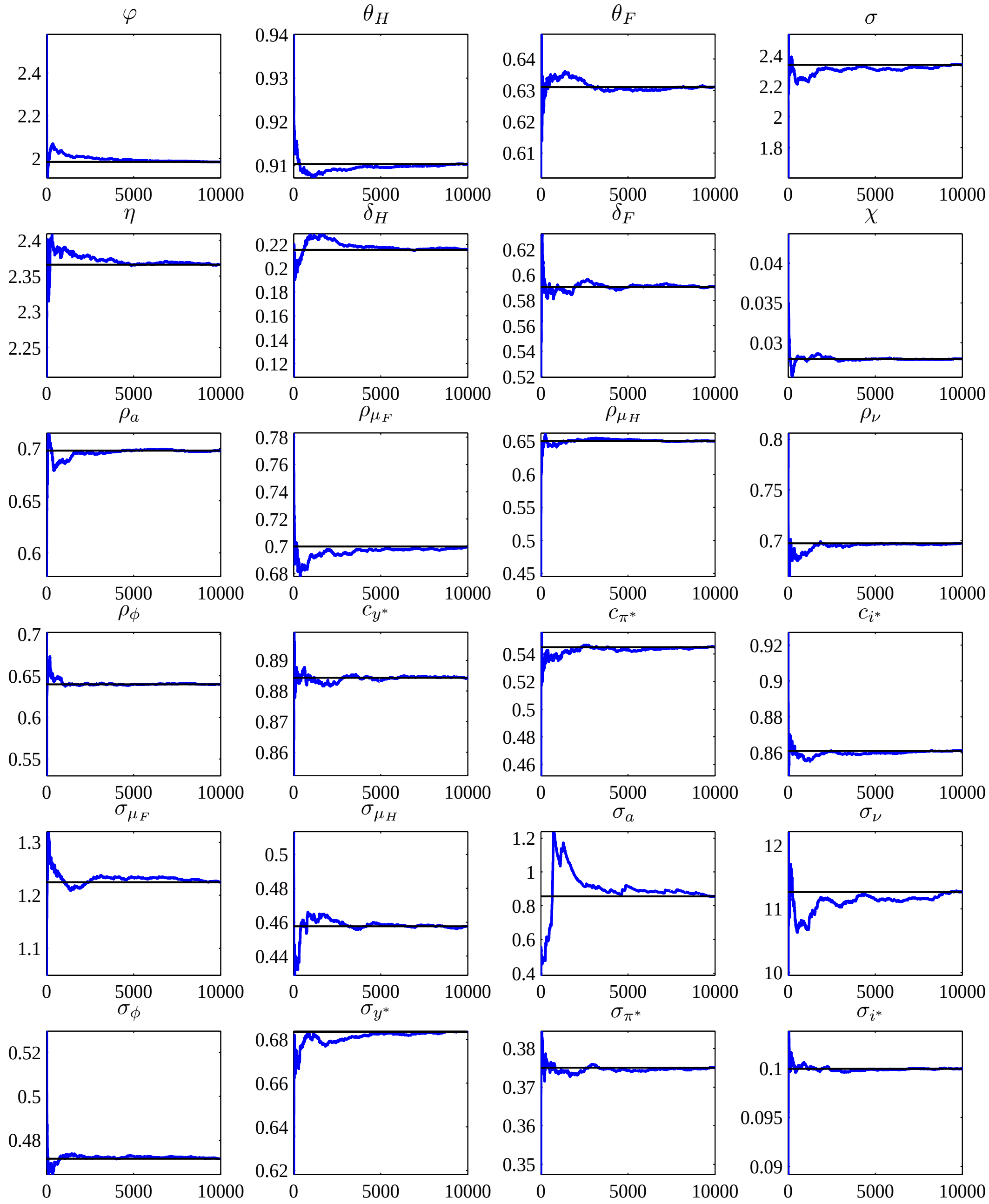


Figure 9: Recursive means of $\mathcal{M}_1$ (no Markov-Switching). Black line indicates the posterior mean.

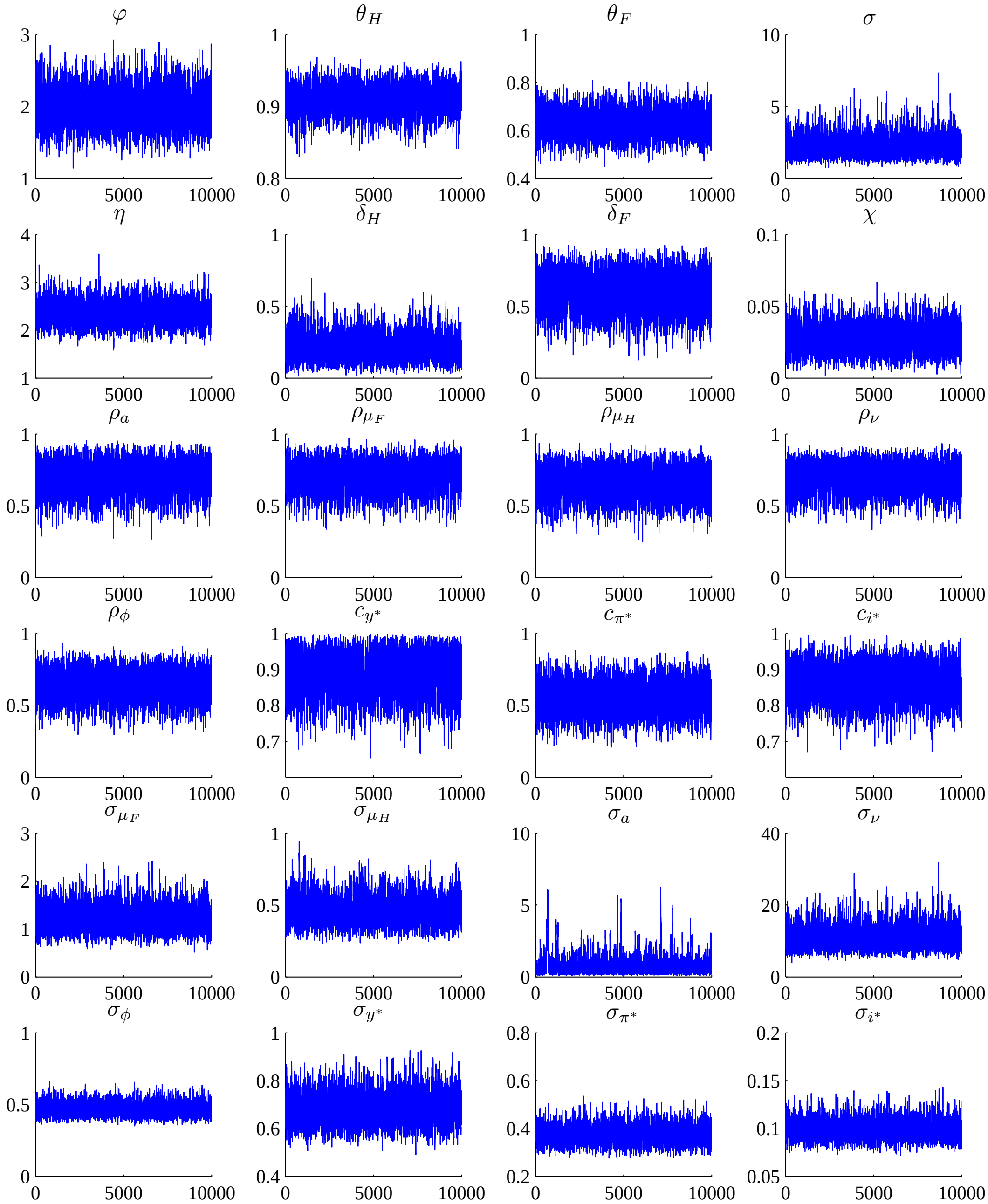


Figure 10: Trace plots of $\mathcal{M}_1$ (no Markov-Switching)

A4. $\mathcal{M}_2$: Convergence diagnostics: figures and tables

Table 7: Left: Autocorrelation among the draws, based on a sample of 10000. Right: Raferty-Lewis convergence diagnostics with q=0.025, r=0.1, s=0.95. I-stat should be less than 5.

	Lag 1.	Lag 5	Lag 10	Lag 50
p_{11}	0.616	0.131	0.014	0.001
p_{22}	0.548	0.072	–0.002	–0.008
φ	0.593	0.071	–0.018	0.001
θ_H	0.640	0.139	0.024	0.011
θ_F	0.612	0.135	0.063	–0.002
σ	0.650	0.175	0.064	0.031
η	0.583	0.067	–0.004	–0.021
δ_H	0.602	0.111	0.033	–0.006
δ_F	0.597	0.120	0.022	0.016
χ	0.632	0.144	0.024	–0.008
ρ_a	0.568	0.036	–0.016	0.018
ρ_{μ_F}	0.582	0.085	–0.011	–0.018
ρ_{μ_H}	0.580	0.086	–0.006	0.005
ρ_ν	0.607	0.116	0.026	0.027
ρ_ϕ	0.556	0.047	–0.004	–0.013
c_{y^*}	0.581	0.075	–0.002	0.030
c_{π^*}	0.577	0.074	0.031	–0.018
c_{i^*}	0.600	0.086	0.034	0.009
σ_{μ_F}	0.667	0.187	0.055	–0.012
σ_{μ_H}	0.668	0.195	0.086	0.051
σ_a	0.976	0.920	0.874	0.584
σ_ν	0.675	0.214	0.087	0.036
σ_ϕ	0.636	0.152	0.042	–0.013
σ_{y^*}	0.602	0.098	0.020	–0.019
σ_{π^*}	0.619	0.100	0.035	–0.004
σ_{i^*}	0.613	0.089	–0.013	–0.010
σ_ϕ	0.508	0.058	–0.004	0.002

	Thin	Burn	Total(N)	(Nmin)	I-stat
p_{11}	2.000	14.000	3642.000	937.000	3.887
p_{22}	2.000	14.000	3642.000	937.000	3.887
φ	2.000	14.000	3642.000	937.000	3.887
θ_H	2.000	14.000	3642.000	937.000	3.887
θ_F	2.000	14.000	3642.000	937.000	3.887
σ	2.000	14.000	3642.000	937.000	3.887
η	2.000	14.000	3642.000	937.000	3.887
δ_H	2.000	14.000	3642.000	937.000	3.887
δ_F	2.000	14.000	3642.000	937.000	3.887
χ	2.000	14.000	3642.000	937.000	3.887
ρ_a	2.000	14.000	3642.000	937.000	3.887
ρ_{μ_F}	2.000	14.000	3642.000	937.000	3.887
ρ_{μ_H}	2.000	14.000	3642.000	937.000	3.887
ρ_ν	2.000	14.000	3642.000	937.000	3.887
ρ_ϕ	2.000	14.000	3642.000	937.000	3.887
c_{y^*}	2.000	14.000	3642.000	937.000	3.887
c_{π^*}	2.000	14.000	3642.000	937.000	3.887
c_{i^*}	2.000	14.000	3642.000	937.000	3.887
σ_{μ_F}	2.000	14.000	3642.000	937.000	3.887
σ_{μ_H}	2.000	14.000	3642.000	937.000	3.887
σ_a	2.000	14.000	3642.000	937.000	3.887
σ_ν	2.000	14.000	3642.000	937.000	3.887
σ_ϕ	2.000	14.000	3642.000	937.000	3.887
σ_{y^*}	2.000	14.000	3642.000	937.000	3.887
σ_{π^*}	2.000	14.000	3642.000	937.000	3.887
σ_{i^*}	2.000	14.000	3642.000	937.000	3.887
σ_ϕ	2.000	14.000	3642.000	937.000	3.887

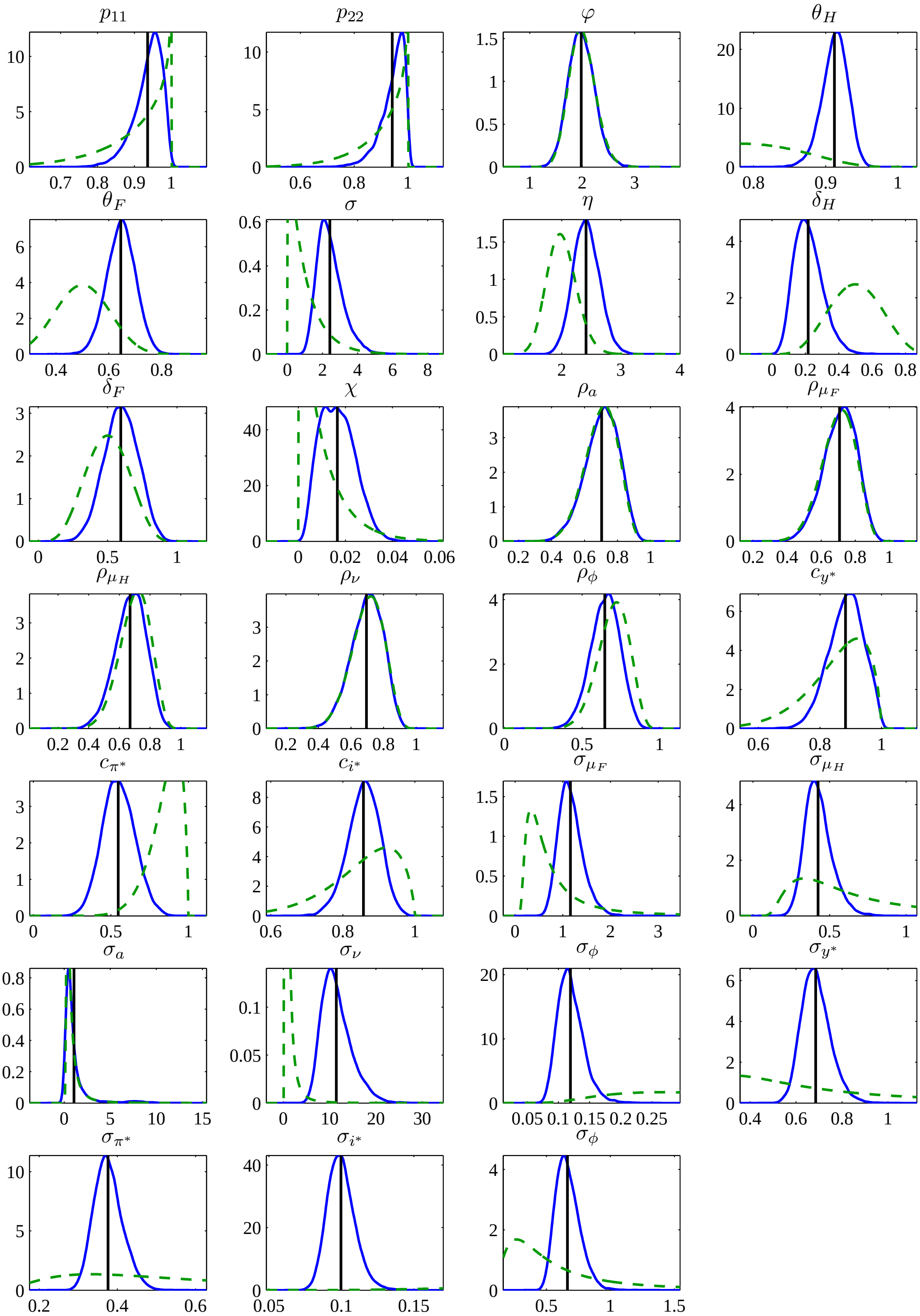


Figure 11: Prior (dashed) and posterior (solid) distributions of $\mathcal{M}_2$

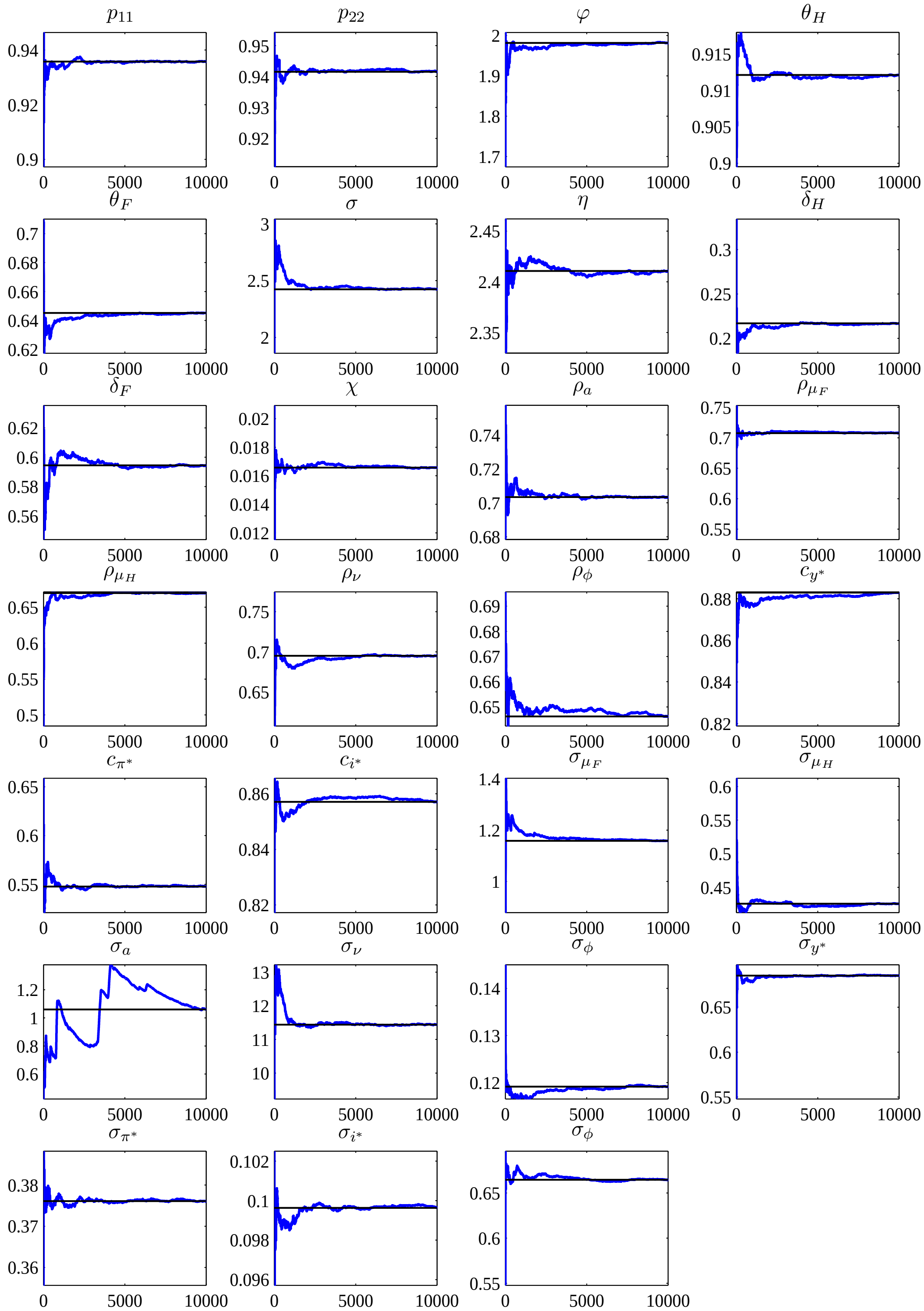


Figure 12: Recursive means of $\mathcal{M}_2$. Black line indicates the posterior mean

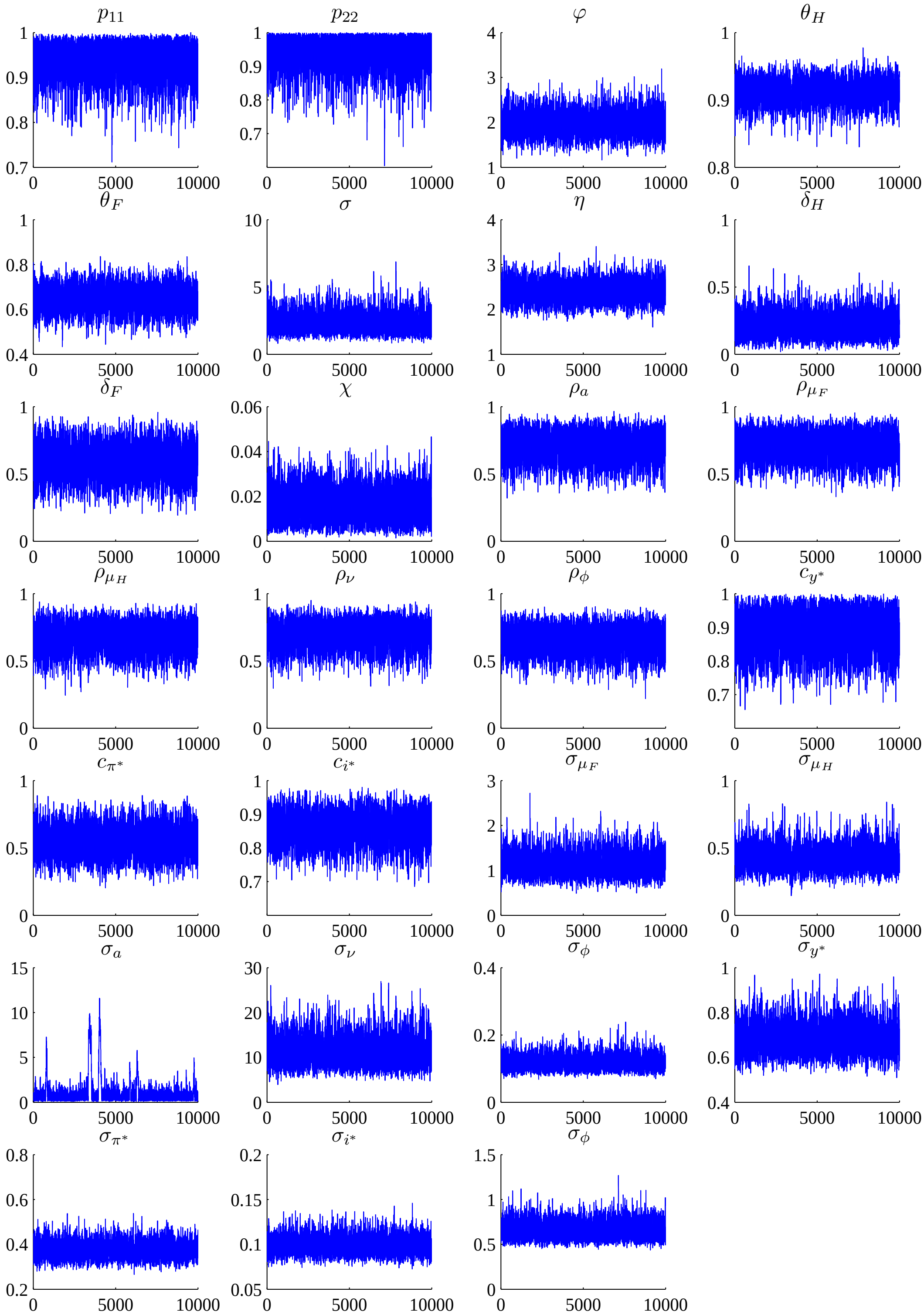


Figure 13: Trace plots of $\mathcal{M}_2$.

A5. Variance decomposition tables

Table 8: Forecast error variance decomposition of consumption for selected periods. State 1: $\sigma_\phi(low)$, in per cent.

	ε_{y^*}	ε_{π^*}	ε_{i^*}	ε_{μ_F}	ε_{μ_H}	ε_a	ε_ν	ε_ϕ
1	0.06	0.72	0.13	0.40	5.70	0.03	92.87	0.09
4	0.11	1.00	0.16	0.25	14.96	0.09	83.36	0.07
8	0.15	1.12	0.15	0.34	23.40	0.14	74.64	0.06
12	0.16	1.14	0.14	0.69	26.59	0.16	71.06	0.06
20	0.17	1.13	0.14	1.04	28.21	0.17	69.08	0.06
40	0.18	1.13	0.15	1.13	28.55	0.17	68.64	0.06
∞	0.18	1.13	0.15	1.13	28.55	0.17	68.63	0.06

Table 9: Forecast error variance decomposition of consumption for selected periods. State 2: $\sigma_\phi(high)$, in per cent.

	ε_{y^*}	ε_{π^*}	ε_{i^*}	ε_{μ_F}	ε_{μ_H}	ε_a	ε_ν	ε_ϕ
1	0.06	0.71	0.13	0.39	5.56	0.03	90.54	2.59
4	0.10	0.98	0.15	0.25	14.65	0.08	81.61	2.16
8	0.14	1.10	0.14	0.34	22.96	0.13	73.24	1.94
12	0.16	1.12	0.14	0.68	26.11	0.15	69.79	1.85
20	0.17	1.11	0.14	1.02	27.72	0.16	67.88	1.79
40	0.18	1.11	0.14	1.11	28.06	0.17	67.46	1.77
∞	0.18	1.11	0.15	1.11	28.06	0.17	67.46	1.77

Table 10: Forecast error variance decomposition of inflation for selected periods. State 1: $\sigma_\phi(low)$, in per cent.

	ε_{y^*}	ε_{π^*}	ε_{i^*}	ε_{μ_F}	ε_{μ_H}	ε_a	ε_ν	ε_ϕ
1	0.03	5.31	0.00	73.85	19.33	0.09	1.38	0.00
4	0.03	7.85	0.00	74.69	16.24	0.08	1.10	0.00
8	0.03	7.10	0.00	75.69	15.85	0.08	1.26	0.00
12	0.03	6.83	0.00	75.38	16.35	0.08	1.32	0.00
20	0.03	6.80	0.00	75.28	16.47	0.08	1.32	0.00
40	0.03	6.80	0.00	75.27	16.47	0.08	1.34	0.00
∞	0.03	6.80	0.00	75.26	16.47	0.08	1.34	0.00

Table 11: Forecast error variance decomposition of inflation for selected periods. State 2: $\sigma_\phi(high)$, in per cent.

	ε_{y^*}	ε_{π^*}	ε_{i^*}	ε_{μ_F}	ε_{μ_H}	ε_a	ε_ν	ε_ϕ
1	0.03	5.30	0.00	73.83	19.32	0.09	1.38	0.03
4	0.03	7.85	0.00	74.68	16.23	0.08	1.10	0.03
8	0.03	7.10	0.00	75.66	15.84	0.08	1.26	0.03
12	0.03	6.83	0.00	75.35	16.35	0.08	1.32	0.03
20	0.03	6.80	0.00	75.26	16.46	0.08	1.32	0.03
40	0.03	6.80	0.00	75.24	16.47	0.08	1.34	0.03
∞	0.03	6.80	0.00	75.24	16.47	0.08	1.34	0.03

Table 12: Forecast error variance decomposition of output for selected periods. State 1: $\sigma_\phi(low)$, in per cent.

	ε_{y^*}	ε_{π^*}	ε_{i^*}	ε_{μ_F}	ε_{μ_H}	ε_a	ε_ν	ε_ϕ
1	1.07	5.88	0.02	6.51	43.62	0.22	42.64	0.04
4	0.41	4.07	0.01	8.34	76.00	0.40	10.75	0.01
8	0.28	3.39	0.01	6.17	82.23	0.46	7.45	0.01
12	0.27	3.26	0.01	6.37	82.53	0.47	7.09	0.01
20	0.29	3.22	0.01	6.96	81.67	0.46	7.37	0.01
40	0.30	3.20	0.01	7.07	81.27	0.46	7.68	0.01
∞	0.30	3.20	0.01	7.07	81.26	0.46	7.69	0.01

Table 13: Forecast error variance decomposition of output for selected periods. State 2: $\sigma_\phi(high)$, in per cent.

	ε_{y^*}	ε_{π^*}	ε_{i^*}	ε_{μ_F}	ε_{μ_H}	ε_a	ε_ν	ε_ϕ
1	1.06	5.81	0.02	6.43	43.09	0.22	42.13	1.24
4	0.41	4.06	0.01	8.32	75.77	0.40	10.72	0.31
8	0.28	3.38	0.01	6.16	82.06	0.46	7.44	0.21
12	0.27	3.25	0.01	6.36	82.37	0.47	7.07	0.20
20	0.29	3.22	0.01	6.95	81.51	0.46	7.36	0.21
40	0.30	3.19	0.01	7.06	81.11	0.46	7.67	0.21
∞	0.30	3.19	0.01	7.06	81.10	0.46	7.68	0.21

Table 14: Forecast error variance decomposition of the interest rate for selected periods.
State 1: $\sigma_\phi(low)$, in per cent.

	ε_{y^*}	ε_{π^*}	ε_{i^*}	ε_{μ_F}	ε_{μ_H}	ε_a	ε_ν	ε_ϕ
1	0.24	0.01	30.44	0.23	0.16	0.00	5.73	63.19
4	0.58	0.23	23.93	7.70	6.70	0.03	28.36	32.47
8	0.62	0.55	12.15	16.06	19.16	0.10	37.36	14.00
12	0.59	0.69	8.07	16.96	26.38	0.14	38.09	9.08
20	0.58	0.76	5.97	16.07	31.98	0.18	37.76	6.70
40	0.58	0.78	5.51	15.60	33.66	0.19	37.53	6.16
∞	0.58	0.78	5.50	15.58	33.69	0.19	37.53	6.15

Table 15: Forecast error variance decomposition of the interest rate for selected periods.
State 2: $\sigma_\phi(high)$, in per cent.

	ε_{y^*}	ε_{π^*}	ε_{i^*}	ε_{μ_F}	ε_{μ_H}	ε_a	ε_ν	ε_ϕ
1	0.01	0.00	1.52	0.01	0.01	0.00	0.29	98.16
4	0.05	0.02	2.22	0.71	0.62	0.00	2.63	93.74
8	0.12	0.11	2.33	3.08	3.67	0.02	7.16	83.52
12	0.16	0.18	2.16	4.54	7.06	0.04	10.20	75.66
20	0.19	0.25	1.98	5.32	10.59	0.06	12.51	69.09
40	0.20	0.27	1.93	5.46	11.78	0.07	13.14	67.15
∞	0.20	0.27	1.93	5.46	11.81	0.07	13.15	67.11

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