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SOME EXAMPLES AND EXPLANATIONS ABOUT THE STRUCTURE OF GRAPHS

Contents

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Abstract: Presented a way for recognition the structure of graphs with exactness up to orbits (positions), isomorphism and other structural properties. It is realized in the form of specific models that are essential attributes for studying the structure of graphs.

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Introductive explanations

It is a much talk about the structure of graph, but what it meant, has remained vague. Structure is an ordering, organizational or constructional side of systemic objects. Structure is classically defined as a permanently associated status of its elements [14, 17]. We demonstrate that the important properties of the structure of a graph are these which allow one structure to distinguish from the other.

Here is presented a way for recognition the structure of the graphs with exactness up to orbits (positions), isomorphism and other structural properties. All the properties of structure are described in corresponding papers [20, 23, 26, 32, 36, 37]. In this introduction give only a summary of main definitions and characteristics for understanding of examples and its explanations.

Recognition of the structure is based on the *identification the relationships between elements* and *their "positions" in the structure*.

1. A primary way for identification the relationships between elements (vertices):
   1.1. Each *binary relation (vertex pair* ***ij****)* can be characterized as an *intersection of surroundings* $\boldsymbol{N_i \cap N_j}$ of the elements (vertices) that is presented in the form of a *binary graph* $\boldsymbol{g_{ij}}$.
   1.2. The *invariants* of $\boldsymbol{g_{ij}}$, where ***+d*** is *collateral-* and ***–d*** *custom distance*, ***n*** *– number of vertices* and ***m*** *– number of edges* identify the binary relation, called *binary sign* $\boldsymbol{\pm d.n.m._{ij}}$.
   1.3. Lexicographically *ordered system of binary signs* – ***structure's model*** **SM** – identifies the *relationships between the elements*, as well as *positions* [15].
   1.4. The positions are *equivalence classes* that on the aspect of group theory to *orbits* are called.

In common case is the structure recognizable by these preliminary (basic) binary signs.

On the other hand is obvious that a large part of the binary signs *are not complete identifiers* of element pairs. Many large symmetric structures require *complementary identification* of binary signs. For this are some ways.

2. Ways of complementary identification the binary signs:
   2.1. Using the complementary binary signs $\boldsymbol{dnm_{ij}^m}$ of the ***high degree m*** binary graphs $\boldsymbol{g_{ij}^m}$, i.e. binary graphs, that remain between elements ***i*** and ***j*** of ***G*** after removing the preliminary binary graph $\boldsymbol{g_{ij}}$ (example 4.3).
   2.2. Using the complementary binary signs of ***local structure models*** $\mathbf{SM}_{ij}$ of binary graphs $\boldsymbol{g_{ij}}$ (example 4.3).
   2.3. Using the complementary binary signs of ***sign structures*** $\boldsymbol{GS_p}$, that consists of element pairs with a certain class of pair signs, independently from their positions (example 1.7).
   2.4. Using complementary binary signs of the ***product of adjacency matrix*** $\boldsymbol{E \times E \times E \times \ldots = E^n}$ where up to certain degree ***n*** the ***values*** of elements $\boldsymbol{e^n_{ij}}$ as well as the number ***p*** of their ***differences*** increased, and then stopped (examples 1.6 and 4.2).

The meaning of the preliminary binary signs do not lost also in case of perfected binary signs. These characterize the belonging of elements and connections to the paths and girths that are need for treatment of the structure. Perfected binary sign constitutes a quintuplet $\boldsymbol{\pm d.n.m.e^n_{ij}}$, where the last represents the perfecting (examples 1.6, 1.7, 4.2, 4.3).

Structure model **SM** contains all the characteristics of structure that are necessary for identification the structure as a whole and for distinguishing of various structures (chapter 4).

Structure is a *complete invariant of isomorphic graphs*. Isomorphic graphs have equivalent structure (example 4.1).

Essential structural properties are ***regularity*** and ***symmetry*** Regularity and symmetry of structure are very rare conditions, but for that reason are more intriguing. With the symmetry of the graphs arise confusions. Some call to symmetric the simple graphs, because the edges are not directed. Others call symmetric the transitive of vertices or edges graphs that mean the transitivity domain of automorphisms in ***AutG***. With the latter must be consent.

To the assumption of symmetry is ***regularity***, but not vice versa. Regularities are several, and they are easily readable out from structure model **SM**. We define these.

3. Kinds of regularity:
   - 3.1. Structure (graph), where by each element $\boldsymbol{i}$ the number $\boldsymbol{v}$ of binary(+)signs $+\boldsymbol{dnm_{ij}}$ is constant is ***v-degree-regular***.
   - 3.2. Degree-regular structure (graph), where by each element $\boldsymbol{i}$ the number of partial signs $-\boldsymbol{d}$ of binary(–)signs $-\boldsymbol{dnm_{ij}}$ are constant is ***d-distance-regular.***
   - 3.3. Degree-regular structure (graph), where by each element $\boldsymbol{i}$ the number of partial signs $+\boldsymbol{d}$ of binary(+)signs $+\boldsymbol{dnm_{ij}}$ are constant is ***(d+1)-girth-regular***.
   - 3.4. Degree-regular structure (graph), where each element $\boldsymbol{i}$ belong to a clique with the power $\boldsymbol{n}<|\boldsymbol{V}|$ is ***n-clique-regular***, where $|\boldsymbol{V}|$ is the number of structural elements.
   - 3.5. Degree-regular structure (graph), where each pair of adjacent elements holds $\boldsymbol{a}\geq 0$ common neighbors and each nonadjacent pair holds $\boldsymbol{b}\geq 1$ common neighbors is ***strongly-regular***.

Symmetry is a structural property that expressed as recurrence of similar elements (particles) in the space or time [14, 17]. Indeed, what greater are the positions and what smaller is the number of positions then greater the structural symmetry. Symmetry is measurable. Symmetry of the structure depends on the number and size of positions. We define these.

4. Kinds of symmetry:
   - 4.1. *Complete* structure (graph) has *one* element- $\boldsymbol{\Omega V_k}$ and *one* binary position $\boldsymbol{\Omega R_n}$ and it is ***completely symmetric.***
   - 4.2. *Transitive* structure (graph), as it in graph theory called, has *one* element position $\boldsymbol{\Omega V_k}$ and it is ***element symmetric***.
   - 4.3. *Element symmetric* structure (graph), that has *one* binary(+)position („edge position“) $\boldsymbol{\Omega R_n^+}$ and *one* binary(–)position („non-edge position“) $\boldsymbol{\Omega R_n^-}$ is ***bisymmetric*** (examples 1.1 and 1.2).
   - 4.4. *Element symmetric* structure (graph), that has *one* binary(+)position $\boldsymbol{\Omega R_n^+}$ and *several* binary(–)positions $\boldsymbol{\Omega R_n^-}$ is (+)***symmetric*** or ***edge symmetric*** (examples 1.3 and 1.4).
   - 4.5. *Element symmetric* structure (graph), that has *several* binary(+)- $\boldsymbol{\Omega R_n^+}$ and *several* binary(–)positions $\boldsymbol{\Omega R_n^-}$ is ***poly-symmetric*** (examples 1.6 and 1.7).
   - 4.6. Structure (graph), that *not element symmetric*, but has *one* binary(+)position $\boldsymbol{\Omega R_n^+}$ is ***semi-symmetric*** (example 1.5)**.**
   - 4.7. Structure (graph), that *not element symmetric*, i.e. that has *more than one* element positions, with *at least* one of these positions $\boldsymbol{\Omega V_k}$ has *at least* two elements is ***partially symmetric*** (examples 2.1 – 2.5, 3.1 – 3.10, 5.1 and 5.2).
   - 4.8. Structure (graph), where the number $\boldsymbol{K}$ of element positions $\boldsymbol{\Omega V_k}$ equals to the number $|V|$ of elements (vertices) is ***0-symmetric*** or ***completely asymmetric*** (example 3.11 and 5.3b). Almost all of the random graphs are *0*-symmetric.

Symmetry and regularity is pretty related to with each other. For example, all the *element symmetric (transitive)* graphs are *girth regular* or *clique regular* (examples 1.1 – 1.6), and all the connected *bisymmetric* graphs are *strongly regular* (examples 1.1 and 1.2), etc.

Each position can be “naturalized” in the form of a ***position structure*** $\boldsymbol{GS_n}$.

5. Properties of position structures:
   - 5.1. Position structure $\boldsymbol{GS_n}$ is a structure that consists of element pairs, which belong to a certain *binary position* $\boldsymbol{\Omega R_n}$. The number of position structures equal to the number of binary positions (example 1.5).
   - 5.2. Position structure is *element symmetric*, i.e. its elements belong to the same position $\boldsymbol{\Omega V_{k=1}}$.
   - 5.3. To the binary(+)position $\boldsymbol{\Omega R_n^+}$ corresponds a position(+)structure $\boldsymbol{GS_n^+}$ is a *partial structure* of $\boldsymbol{GS}$; to the binary(–)position $\boldsymbol{\Omega R_n^-}$ corresponds a position(–)structure $\boldsymbol{GS_n^-}$ is a *partial structure of complement* $\boldsymbol{\urcorner GS}$.

5.4. Some position structure $\boldsymbol{GS_n}$ can be *isomorphic with initial structure,* $\boldsymbol{GS}$, $\boldsymbol{GS_n \cong GS}$ (for example, an position structure of the cube is also cube).
5.5. *Different position structures* $\boldsymbol{GS_n}$ of initial structure $\boldsymbol{GS}$ or *position structures of different structures* can be *isomorphic* or *coincides*.

The position structures are needed for opening various "hidden sides" of structure.

For obtaining complementary information about structural properties *is sufficient to look* the works [32, 35, 37], that are simply obtainable also in digital form.

Essential are also the properties of ***elementary structural changes***.

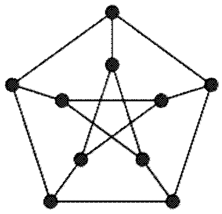
6. Properties of elementary structural changes:
   6.1. By removing an edge $\boldsymbol{G \backslash e_{ij}}$ of $\boldsymbol{G}$ obtained a *greatest subgraph* $\boldsymbol{G^{sub}}$**.**
   6.2. With adding an edge $\boldsymbol{G \cup e_{ij}}$ to $\boldsymbol{G}$ obtained a *smallest supergraph* $\boldsymbol{G^{sup}}$.
   6.3. The number of $\boldsymbol{G^{sub}}$ equals to the number of edges and the number of $\boldsymbol{G^{sup}}$ to number of "non-edges.
   6.4. Greatest subgraphs $\boldsymbol{G^{sub}}$ and smallest supergraphs $\boldsymbol{G^{sup}}$ called ***adjacent graphs*** $\boldsymbol{G^{adj}}$ of graph $\boldsymbol{G}$**.**
   6.5. If the adjacent graphs $\boldsymbol{G^{adj}}$ are obtained on the ground of the same binary position $\boldsymbol{\Omega R_n}$ then are these *isomorphic* and constitute an ***adjacent structure*** $\boldsymbol{GS^{adj}_n}$ (examples 5.2 and 5.3).
   6.6. Disjunctive edge operation $\boldsymbol{F_n}=\{(\boldsymbol{f_{ij}})_{\boldsymbol{1}} \vee \ldots \vee (\boldsymbol{f_{ij}})_{\boldsymbol{q}}\}_{\boldsymbol{n}}$ that changes the structure $\boldsymbol{GS}$ to its adjacent structure $\boldsymbol{GS^{adj}_n}$ called ***morphism*** $\boldsymbol{F_n}$, $\boldsymbol{F_n: GS \rightarrow GS^{adj}_n}$.

Here is essential, that morphisms and adjacent structures are related with the ***reconstruction problem*** (chapter 5). All the graphs with $\boldsymbol{n}$ vertices form a ***system of adjacent structures*** [32, 35, 37]**.**

By studying the graph-structure is useful to treat also its *complement*, since it helps to recognize its properties. In following presented examples with explanations the enable to study the essential properties of structure.

1. Structures of known symmetric graphs

Example 1.1. Petersen graph ***Pet*** (the numbering starts from the upper element and goes clockwise), the structure model for ***Pet*** and its complement ***PetC***:



`A:-2.3.2; B:+4.10.15.`

1	1	1	1	1	1	1	1	1	1		u_i	
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	***i***	*AB*	***k***
0	**+B**	*A*	*A*	**+B**	+*B*	*A*	*A*	*A*	*A*	**1**	*63*	1
	0	**+B**	*A*	*A*	*A*	**+B**	*A*	*A*	*A*	**2**	*63*	1
		0	**+B**	*A*	*A*	*A*	**+B**	*A*	*A*	**3**	*63*	1
			0	**+B**	*A*	*A*	*A*	**+B**	*A*	**4**	*63*	1
				0	*A*	*A*	*A*	*A*	**+B**	**5**	*63*	1
					0	*A*	**+B**	+*B*	*A*	**6**	*63*	1
						0	*A*	**+B**	**+B**	**7**	*63*	1
							0	*A*	**+B**	**8**	*63*	1
								0	*A*	**9**	*63*	1
									0	**10**	*63*	1

`A:-2.6.12; B:+2.5.8.`

1	1	1	1	1	1	1	1	1	1	u_i
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	*AB*
0	*A*	**+B**	+*B*	*A*	*A*	**+B**	**+B**	**+B**	**+B**	*36*
	0	*A*	**+B**	**+B**	**+B**	*A*	**+B**	**+B**	**+B**	*36*
		0	*A*	**+B**	**+B**	**+B**	*A*	**+B**	**+B**	*36*
			0	*A*	**+B**	**+B**	**+B**	*A*	**+B**	*36*
				0	**+B**	**+B**	**+B**	**+B**	*A*	*36*
					0	**+B**	*A*	*A*	**+B**	*36*
						0	**+B**	*A*	*A*	*36*
							0	**+B**	*A*	*36*
								0	**+B**	*36*
									0	*36*

Structural properties to show that is possible to read out from the structure model:

a) Petersen graph ***Pet*** has two binary positions, i.e. it is ***bisymmetric***. Thus, it has *two adjacent structures* GS^{adj}_n in the form of *one greatest sub-structure* $GS^{sub}_{n=+B}$ (reflected as its 15 possible isomorphic greatest sub-graphs) and *one smallest superstructure* $GS^{sup}_{n=-A}$ (reflected as its 30 possible isomorphic smallest super-graphs).

b) From bisymmetry concludes ***strong regularity*** of ***Pet.***

c) Graph ***Pet*** is ***5-girth-regular***, there exist twelve *5-girths*, in present case: (1)**:** 1-2-3-4-5-1**,** (2)**:** 6-8-10-7-9-6**,** (3)**:** 1-2-3-8-6-1**,** (4)**:** 1-2-7-10-5-1**,** (5)**:** 1-5-4-9-6-1**,** (6)**:** 2-3-4-9-7-2**,** (7)**:** 3-4-5-10-8-3**,** (8)**:** 1-2-7-9-6-1**,** (9)**:** 1-5-10-8-6-1**,** (10)**:** 2-3-8-10-7-2**,** (11)**:** 3-4-9-6-8-3**,** and (12)**:** 4-5-10-7-9-4. Each element belongs to six girths, each edge belongs to four girths.

d) Binary sign ***+4.10.15*** means, that the element pair belongs to an assemblage of 5-girths, which consists of 10 elements and 15 connections (edges) – it is the *complete invariant* of Petersen graph, such sign do not have other structures.

e) The ***complement*** of Petersen graph ***PetC*** is ***4-clique-regular.*** Explicit clique sign do not exist, but *binary graph* of binary sign ***+2.5.8*** contains the *4*-clique. For example, the local structure model of binary graph with sign ***+2.5.8*** for elements 1 and 3 contains the signs of *4*-clique, ***+2.4.6***, that shows the existence of *4*-clique 1,3,9,10:

`A: -2.4.5;  B: +2.3.3;  C: +2.4.6;  D: +2.5.8.`

1	**3**	**9**	**10**	**7**	***i***	*ABCD*	***k***	123
0	***D***	***C***	***C***	***B***	**1**	*0**121***	1	**121**
	0	***C***	***C***	***B***	**3**	*0**121***	1	**121**
		0	***C***	−*A*	**9**	*1**030***	2	**21**0
			0	−*A*	**10**	*1**030***	2	**21**0
				0	**7**	*2**200***	3	**2**00

f) And so exists in the complement five intersected *4*-cliques, in present case with elements: (1)**:** 1,3,9,10**;** (2)**:** 2,4,6,10**;** (3)**:** 1,4,7,8**;** (4)**:** 2,5,8,9**;** and (5)**:** 3,5,6,7. Each element belongs to two cliques where each edge belongs to one clique.

g) Invariants and measures:

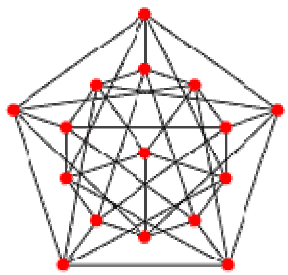
G	|E|	k	N^+	N^-	P	CL	MC	DM	SEV^+	SE^+	SRV	HR	SR	aut
Pet	15	1	1	1	2	2	5	2	**15^1**	*1.000*	**$15^1 30^1$**	*0.2767*	***0.8338***	120
PetC	30	1	1	1	2	4	4	2	**30^1**	*1.000*				

Not every strongly regular graph can be bisymmetric. Among the graphs with up to 20 elements exists 39 *bisymmetric* & *strongly regular* & *clique- or girth-regular* graphs, including the 27 simply constructed *n-m-cliques* and 12 "non-m-n-cliques", to where belongs also Petersen graph. As a rule, the lists of strongly regular graphs are incomplete. By help of the structure models succeeded these lists to supplement.

It is deal with partial coincidence of bisymmetry and strong regularity. Bisymmetry includes also the disconnected structures and strong regularity can be exists in the case of mono-, poly-, and partial symmetry. Although among the structures with up to 20 elements it not been observed. Here has treated only symmetric structures, i.e. graphs that have large positions.

To such part belongs also the Clebsh graph (called also Greewood-Cleason graph) with very interesting structural properties.

<u>Example 1.2.</u> Clebsh graph ***Cle*** (the numbering starts from the upper element and goes clockwise, 16 is in the centre), the structure's model of ***Cle*** and its complement ***CleC***:



A:-2.4.4; ***B:+3.10.13.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	i	AB	k
0	+B	A	A	+B	A	A	+B	A	A	A	+B	A	A	+B	A	1	105	1
	0	+B	A	A	A	+B	A	A	+B	A	A	A	+B	A	A	2	105	1
		0	+B	A	+B	A	A	+B	A	A	+B	A	A	A	A	3	105	1
			0	+B	A	A	+B	A	A	+B	A	A	+B	A	A	4	105	1
				0	+B	A	A	A	+B	A	A	+B	A	A	A	5	105	1
					0	+B	A	A	A	A	A	A	A	+B	+B	6	105	1
						0	+B	A	A	+B	A	+B	A	A	A	7	105	1
							0	+B	A	A	A	A	A	A	+B	8	105	1
								0	+B	A	A	+B	A	+B	A	9	105	1
									0	+B	A	A	A	A	+B	10	105	1
										0	+B	A	A	+B	A	11	105	1
											0	+B	A	A	+B	12	105	1
												0	+B	A	A	13	105	1
													0	+B	+B	14	105	1
														0	A	15	105	1
															0	16	105	1

Structural properties:

- **a)** Graph ***Cle*** is ***bisymmetric*** and has *one greatest sub-structure* $\boldsymbol{GS^{sub}_{n=+B}}$ (reflected as its 40 possible isomorphic greatest subgraphs) and *one smallest superstructure* $\boldsymbol{GS^{sup}_{n=-A}}$ (reflected as its 80 possible isomorphic smallest super-graphs).
- **b)** From bisymmetry ***Cle*** concludes its ***strong regularity.***
- **c)** From binary sign ***+3.10.13*** concludes ***4-girth-regularity*** of ***Cle.***
- **d)** Graph ***Cle*** appear also to ***4-partite*** with incompletely connected parts on *4-elementical bases*. But it is no quadroclique. The parts are *variety*, where, for example one variant is **A=5,8,12,15; B=3,7,10,14; C=1,4,9,16**; and **D=2,6,11,13.**:
- **e)** Binary signs and structure model of ***complement CleC***:

A:-2.8.24; ***B:+2.8.22.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	i	A B	k
0	A	+B	+B	A	+B	+B	A	+B	+B	+B	A	+B	+B	−A	B	1	510	1
	0	A	+B	+B	+B	A	+B	+B	A	+B	+B	+B	A	+B	+B	2	510	1
		0	A	+B	A	+B	+B	A	+B	+B	A	+B	+B	+B	+B	3	510	1
			0	A	+B	+B	A	+B	+B	A	+B	+B	A	+B	+B	4	510	1
				0	A	+B	+B	+B	A	+B	+B	A	+B	+B	+B	5	510	1
					0	A	+B	+B	+B	+B	+B	+B	+B	A	A	6	510	1
						0	A	+B	+B	A	+B	A	+B	+B	+B	7	510	1
							0	A	+B	+B	+B	+B	+B	+B	A	8	510	1
								0	A	+B	+B	A	+B	A	+B	9	510	1
									0	A	+B	+B	+B	+B	A	10	510	1
										0	A	+B	+B	A	+B	11	510	1
											0	A	+B	+B	A	12	510	1
												0	A	+B	+B	13	510	1
													0	A	A	14	510	1
														0	+B	15	510	1
															0	16	510	1

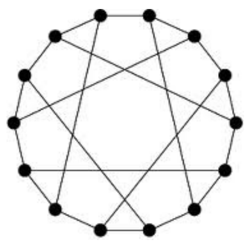
f) From 4-elemintic parts of ***Cle*** concludes the ***4-clique regularity*** of various cliques of complement ***CleC***.

g) On the other hand, in case of each vertex of ***Cle*** exist between its 5 adjacent vertices adjacencies (edges), from which concludes also a ***5-clique-regularity*** of complement ***CleC***. We can in ***CleC*** to find 16 different 5-cliques, such as (beginning at the adjacent vertices of first vertex of ***Cle***) **2,5,8,12,15**; **1,3,7,10,14**; … to ending with **6,8,10,12,14**.

h) Invariants and measures of graph and its complement:

G	|E|	k	N^+	N^-	P	CL	MC	DM	SEV^+	SE^+	SRV	HR	SR
Cle	40	1	1	1	2	2	4	2	40^1	1.000	$40^1 80^1 0$	0.2762	0.8670
CleC	80	1	1	1	2	5	3	2	80^1	1.000		0	

Example 1.3, Heawood graph ***Hea*** (the numbering starts from the upper element and goes clockwise) the structure model of ***Hea*** and its complement ***HeaC***:



A:-3.8.9; B:-2.3.2; ***C:+5.14.21.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	i	ABC	k	deg
0	+C	B	A	B	+C	B	A	B	A	B	A	B	+C	1	463	1	3
	0	+C	B	A	B	A	B	A	B	+C	B	A	B	2	463	1	3
		0	+C	B	A	B	+C	B	A	B	A	B	A	3	463	1	3
			0	+C	B	A	B	A	B	A	B	+C	B	4	463	1	3
				0	+C	B	A	B	+C	B	A	B	A	5	463	1	3
					0	+C	B	A	B	A	B	A	B	6	463	1	3
						0	+C	B	A	B	+C	B	A	7	463	1	3
							0	+C	B	A	B	A	B	8	463	1	3
								0	+C	B	A	B	+C	9	463	1	3
									0	+C	B	A	B	10	463	1	3
										0	+C	B	A	11	463	1	3
											0	+C	B	12	463	1	3
												0	+C	13	463	1	3
													0	14	463	1	3

Structural properties:

- **a)** Graph ***Hea*** has *one* vertex position (is "transitive") and one edge position, by ***+C***. Consequently, ***Hea*** is ***edge symmetric***, but it has also *two binary(–)positions,* by *–A* and *–B* correspondingly. Thus, it has *one greatest sub-structure* $\boldsymbol{GS^{sub}_{n=+C}}$ (reflected as its 21 possible isomorphic greatest subgraphs) and *two smallest superstructures* $\boldsymbol{GS^{sup}_{n=-A}}$ and $\boldsymbol{GS^{sup}_{n=-B}}$ (reflected as its 28 and 42 possible isomorphic smallest super-graphs).
- **b)** From existence of two binary(–)positions concludes existence also of two ***position structures***: 1) $\boldsymbol{GS_{n=-A}}$ with binary signs *A*: *–3.10,16; B: –2.2.4;* ***C: +3.8,10***; 2) $\boldsymbol{GS^{sup}_{n=-B}}$ with binary signs *A: –u.2.0;* ***B:+2.7.21*** that constitutes two separate 7-cliques.
- **c)** Binary sign ***+5.14.21*** mean that element pair and corresponding edge belong to an *assemblage of 6-girths* with 14 vertices and 21 edges. Consequently, ***Hea*** is ***6-girth regular***.
- **d)** From 6-girth regularity concludes that graph ***Hea*** is also ***bipartite***, where its parts in present case divide to vertices with even numbers and vertices with odd numbers.
- **e)** The binary signs and structure model of complement ***HeaC***:

A:-2.10.36; **B:+2.8.22; C:+2.9.30.**

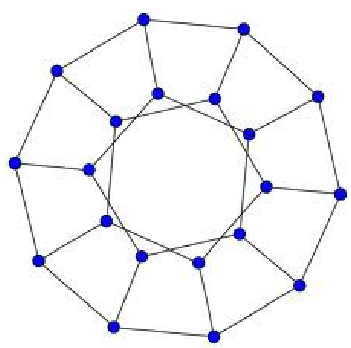
1	2	3	4	5	6	7	8	9	10	11	12	13	14	*i*	*ABC*	*k*	*deg*
0	A	+C	+B	+C	A	+C	+B	+C	+B	+C	+B	+C	A	1	346	1	10
	0	A	+C	+B	+C	+B	+C	+B	+C	A	+C	+B	+C	2	346	1	10
		0	A	+C	+B	+C	A	+C	+B	+C	+B	+C	+B	3	346	1	10
			0	A	+C	+B	+C	+B	+C	+B	+C	A	+C	4	346	1	10
				0	-A	+C	+B	+C	A	+C	+B	+C	+B	5	346	1	10
					0	A	+C	+B	+C	+B	+C	+B	+C	6	346	1	10
						0	A	+C	+B	+C	A	+C	+B	7	346	1	10
							0	A	+C	+B	+C	+B	+C	8	346	1	10
								0	-A	+C	+B	+C	A	9	346	1	10
									0	A	+C	+B	+C	10	346	1	10
										0	A	+C	+B	11	346	1	10
											0	A	+C	12	346	1	10
												0	A	13	346	1	10
													0	14	346	1	10

- **f)** *Complement* ***HeaC*** has two edge positions, by ***+B*** and ***+C***, consequently it is ***poly symmetric.*** From bipartite of ***Hea*** concludes that ***HeaC*** consist of *two mutually connected 7-cliques*, it is ***7-clique regular***, where the cliques correspond to the parts of ***Hea.***
- **g)** Invariants and measures:

G	*|E|*	*k*	N^+	N^-	*P*	*CL*	*MC*	*DM*	SEV^+	SE^+	*SRV*	*HR*	*SR*	*aut*
Hea	21	1	1	2	3	2	6	3	$\mathbf{21^1}$	1.000	$\mathbf{21^1 28^1 42^1}$	0.4595	**0.7655**	336
HeaC	70	1	2	1	3	7	3	2	$\mathbf{28^1 42^1}$	0.7935				

Following known graph is not bipartite but its complement contains interesting clique regularity,

Example 1.4. A diagram of dodecahedron or Hamilton graph ***Ham***, the structure model of ***Ham*** and its complement ***HamC***:



–A=–5.20.30; –B=–4.8.9; –C=–3.4.3; –D=–2.3.2; ***+E=+4.8.9.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	i	ABCD**E**	k
0	**E**	–D	–C	–B	–C	–D	**E**	–D	–C	–B	–A	–B	–C	–D	–D	–C	–C	–D	**E**	1	1366**3**	1
	0	**E**	–D	–C	–C	–D	–D	–C	–B	–A	–B	–C	–D	–**E**	–D	–C	–B	–C	–D	2	1366**3**	1
		0	**E**	–D	–D	**E**	–D	–C	–C	–B	–C	–C	–D	–D	–C	–B	–A	–B	–C	3	1366**3**	1
			0	**E**	–D	–D	–C	–B	–C	–C	–D	–D	**E**	–D	–C	–C	–B	–A	–B	4	1366**3**	1
				0	**E**	–D	–C	–C	–D	–D	**E**	–D	–D	–C	–B	–C	–C	–B	–A	5	1366**3**	1
					0	**E**	–D	–D	**E**	–D	–D	–C	–C	–B	–A	–B	–C	–C	–B	6	1366**3**	1
						0	**E**	–D	–D	–C	–C	–B	–C	–C	–B	–A	–B	–C	–C	7	1366**3**	1
							0	**E**	–D	–C	–B	–A	–B	–C	–C	–B	–C	–D	–D	8	1366**3**	1
								0	**E**	–D	–C	–B	–A	–B	–C	–C	–D	**E**	–D	9	1366**3**	1
									0	**E**	–D	–C	–B	–A	–B	–C	–D	–D	–C	10	1366**3**	1
										0	**E**	–D	–C	–B	–C	–D	**E**	–D	–C	11	1366**3**	1
											0	**E**	–D	–C	–C	–D	–D	–C	–B	12	1366**3**	1
												0	**E**	–D	–D	**E**	–D	–C	–C	13	1366**3**	1
													0	**E**	–D	–D	–C	–B	–C	14	1366**3**	1
														0	**E**	–D	–C	–C	–D	15	1366**3**	1
															0	**E**	–D	–D	**E**	16	1366**3**	1
																0	**E**	–D	–D	17	1366**3**	1
																	0	**E**	–D	18	1366**3**	1
																		0	**E**	19	1366**3**	1
																			0	20	1366**3**	1

–A=–2.16.102; ***+B=+2.14.78; +C=+2.14.79; +D=+2.15.89.***

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	i	A**BCCD**	k
0	–A	D	B	C1	B	D	–A	D	B	C1	C2	C1	B	D	D	B	B	D	–A	1	3**6316**	1
	0	–A	D	B	B	D	D	B	C1	C2	C1	B	D	–A	D	B	C1	B	D	2	3**6316**	1
		0	–A	D	D	–A	D	B	B	C1	B	B	D	D	B	C1	C2	C1	B	3	3**6316**	1
			0	–A	D	D	B	C1	B	B	D	D	–A	D	B	B	C1	C2	C1	4	3**6316**	1
				0	–A	D	B	B	D	D	–A	D	D	B	C1	B	B	C1	C2	5	3**6316**	1
					0	–A	D	D	–A	D	D	B	B	C1	C2	C1	B	B	C1	6	3**6316**	1
						0	–A	D	D	B	B	C1	B	B	C1	C2	C1	B	B	7	3**6316**	1
							0	–A	D	B	C1	C2	C1	B	B	C1	B	D	D	8	3**6316**	1
								0	–A	D	B	C1	C2	C1	B	B	D	–A	D	9	3**6316**	1
									0	–A	D	B	C1	C2	C1	B	D	D	B	10	3**6316**	1
										0	–A	D	B	C1	B	D	–A	D	B	11	3**6316**	1
											0	–A	D	B	B	D	D	B	C1	12	3**6316**	1
												0	–A	D	D	–A	D	B	B	13	3**6316**	1
													0	–A	D	D	B	C1	B	14	3**6316**	1
														0	–A	D	B	B	D	15	3**6316**	1
															0	–A	D	D	–A	16	3**6316**	1
																0	–A	D	D	17	3**6316**	1
																	0	–A	D	18	3**6316**	1
																		0	–A	19	3**6316**	1
																			0	20	3**6316**	1

Structural properties:

- **a)** Graph ***Ham*** is ***edge symmetric*** (has one edge position ***+E*** and four "non-edge" positions, by ***–****A, **–***B, C and D*. Consequently its complement ***HamC*** is ***poly symmetric***.
- **b)** In complement ***HamC*** the explicit clique signs no exist, but in the processing the binary graphs $\boldsymbol{g_{ij}}$, for example with signs ***+B=+2.14.78***, obtained local structure models $\mathbf{SM}_{1.4}$, $\mathbf{SM}_{5.9}$, $\mathbf{SM}_{3.16}$, $\mathbf{SM}_{6.13}$ and $\mathbf{SM}_{5.8}$, contain *8-clique signs* ***+2.8.28***. On the ground of such local structure models can be to recognize all the "hidden" ***partial 8-cliques*** of ***HamC***:

i=	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
I	●			●			●			●		●			●		●		●	
II		●			●		●		●		●			●			●			●
III	●		●			●			●			●		●		●		●		
IV		●		●		●		●			●		●			●			●	
V			●		●			●		●			●		●			●		●

c) Thus, the complement ***HamC*** is ***8-clique-regular***, where all five partial cliques are ***intercrossed***, and where all the 10 intercrossing edges belong to binary position ***C2***.

i-j=	1-12	2-11	3-18	4-19	5-20	6-16	7-17	8-13	9-14	10-15
Partial clique	**I**	**II**	**III**	**I**	**II**	**III**	**I**	**IV**	**II**	**I**
Partial clique	**III**	**IV**	**V**	**IV**	**V**	**IV**	**II**	**V**	**III**	**V**

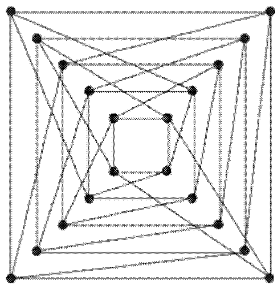
d) Invariants and measures:

G	***|E|***	***k***	***N⁺***	***N⁻***	***P***	***CL***	***MC***	***DM***	***SEV⁺***	***SE⁺***	***SRV***	***HR***	***SR***
Ham	30	1	1	4	5	2	5	5	**30^1**	*1.000*	**$10^1 30^2 60^2$**	*0.6366*	***0.5022***
HamC	160	1	4	1	5	8	3	2	**$10^1 30^1 60^2$**	*0.5590*			

f) The ***position structures*** of ***Ham***: 1) by position *–B* a graph that is *isomorphic* with ***Ham***; 2) by position *–C* structure with binary signs *A:–3.14.30, B: –2.4.4, C:–2.3.2,* ***D: +2.4.6,*** i. e. an *edge symmetric* and *4-clique regular* graph; 3) by *–D* structure with binary signs *A:–3.14.30; B: –2.4.4;* ***C: +2.3.3***, i. e. an *edge symmetric* and *3-girth regular* graph.

Form known graphs are *clique regular* also complements of Coxeter's, Folkman's and other graphs. Their originals are bipartite and by all the nature laws represent the complements of such parts self-evidently cliques.

<u>Example 1.5.</u> Folkman graph ***Fol*** (the numbering starts from the main diagonal), its structure model and list of its ***position structures GS_n***:



A:-4.14.21; B:-3.8.10; C:-2.6.8; D:-2.4.4; E:-2.3.2; ***F:+3.6.8.***

1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2		u_i	k	s_i
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	***i***	*ABCDE**F***		12
0	-E	-E	-E	-E	-E	-E	-E	-E	-C	**F**	-B	**F**	-B	-B	-B	-B	-B	**F**	**F**	**11**	*06108**4***	1	0**4**
	0	-E	-E	-E	-E	-E	-E	-C	-E	-B	**F**	-B	**F**	-B	-B	-B	**F**	**F**	-B	**12**	*06108**4***	1	0**4**
		0	-E	-E	-E	-E	-C	-E	-E	-B	-B	**F**	-B	**F**	-B	**F**	**F**	-B	-B	**13**	*06108**4***	1	0**4**
			0	-E	-E	-C	-E	-E	-E	**F**	-B	-B	**F**	-B	**F**	**F**	-B	-B	-B	**14**	*06108**4***	1	0**4**
				0	-C	-E	-E	-E	-E	-B	**F**	-B	-B	**F**	**F**	-B	-B	-B	**F**	**15**	*06108**4***	1	0**4**
					0	-E	-E	-E	-E	-B	**F**	-B	-B	**F**	**F**	-B	-B	-B	**F**	**16**	*06108**4***	1	0**4**
						0	-E	-E	-E	**F**	-B	-B	**F**	-B	**F**	**F**	-B	-B	-B	**17**	*06108**4***	1	0**4**
							0	-E	-E	-B	-B	**F**	-B	**F**	-B	**F**	**F**	-B	-B	**18**	*06108**4***	1	0**4**
								0	-E	-B	**F**	-B	**F**	-B	-B	-B	**F**	**F**	-B	**19**	*06108**4***	1	0**4**
									0	**F**	-B	**F**	-B	-B	-B	-B	-B	**F**	**F**	**20**	*06108**4***	1	0**4**
										0	-A	-D	-D	-A	-D	-D	-A	-D	-D	**1**	*36060**4***	2	**4**0
											0	-A	-D	-D	-D	-A	-D	-D	-D	**2**	*36060**4***	2	**4**0
												0	-A	-D	-A	-D	-D	-D	-D	**3**	*36060**4***	2	**4**0
													0	-A	-D	-D	-D	-D	-A	**4**	*36060**4***	2	**4**0
														0	-D	-D	-D	-A	-D	**5**	*36060**4***	2	**4**0
															0	-D	-A	-A	-D	**6**	*36060**4***	2	**4**0
																0	-D	-A	-A	**7**	*36060**4***	2	**4**0
																	0	-D	-A	**8**	*36060**4***	2	**4**0
																		0	-D	**9**	*36060**4***	2	**4**0
																			0	**10**	*36060**4***	2	**4**0

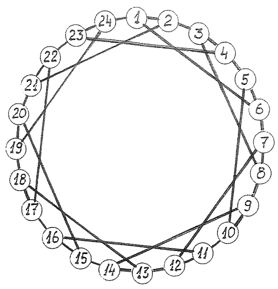
Structural properties:

- **a)** It is know that Folkman graph ***Fol*** is ***4-degree regular, 4-girth regular, bipartite*** and ***semi-symmetric.***
- **b)** From bipartite of ***Fol*** concludes that its ***complement FolC*** consist of *two mutually connected 10-cliques*, it is ***10-clique regular***, where the cliques correspond to the parts of ***Fol.***
- **c)** Graph ***Fol*** includes *six binary positions by –A, –B, –C, –D, –E,* ***+F*** and can be decomposed to six ***position-structures***:
- **d)** To binary position with vertex pairs *–A* corresponds *position structure* $\boldsymbol{Fol}_{n:-A}$ is ***Petersen's graph*(!)**. This fact is showed in partial model $\mathbf{SM}_{2.2}$, if there the sign *–A* replace with Petersen sign ***+4.10.15*** and *–D* replace with sign *–2.3.2* then it is equivalent with structure model of Petersen graph (example 1.1).
- **e)** To binary position *–B* corresponds *position structure* $\boldsymbol{Fol}_{n=-B}$ turns out to *another semi-symmetric graph*, designed by V. Titov [39] that has also a position structure in the form of *Petersen graph*.
- **f)** To binary position *–C* corresponds *position structure* $\boldsymbol{Fol}_{n=-C}$ is a graph with ten *components* of *2-cliques*.
- **g)** To binary position *–D* corresponds *position structure* $\boldsymbol{Fol}_{n=-D}$ is the *complement of Petersen graph* (!).
- **h)** To binary position *–E* corresponds *position structure* $\boldsymbol{Fol}_{n=-E}$ is the *complement of position structure* $\boldsymbol{Fol}_{n=-C}$, i.e. *2-quinta clique*.
- **i)** To binary position ***+F*** corresponds *position structure* $\boldsymbol{Fol}_{n=+F}$ is naturally *Folkman graph* self.

The position structures opens some various "hidden sides" of the structure, that sometimes also "mystical" seems. In principle, the position structures are inevitable, so as the cowering, cliques and others structural attributes, where their identification to a very practical and necessary deemed.

It is obvious that a large part of the binary signs *are not complete invariants* of element pairs. Some of large symmetric structures require a ***perfection*** of binary signs. There exist four ways (see 2.1 – 2.4 in introduction).

<u>Example 1.6.</u> ***Polysymmetric*** **g**raph ***Tev*** and its initial structure model. There we will perfect it by *product perfection* (see 2.4 in introduction) and by *sign structures* (see 2.3 in introduction) for recognition of all the *binary positions*:



A:–5.18.23; B:–4.9.10; C:–4.8.8; D:–4.7.7; E:–3.8.9; F: –3–3–6; G:–3.4.3; H:–2.3.2;
I:+5.10.12; J:+5.12.15; K:+5.14.18.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	*i*	*ABCDEFGH*IJK	*k*
0	**+K**	H	E	H	**+I**	H	F	B	G	D	G	C	A	D	A	B	G	H	F	H	F	H	**+J**	1	22121336**111**	1
	0	**+J**	H	F	H	F	H	G	B	A	D	A	C	G	D	G	B	F	H	**+I**	H	E	H	2	22121336**111**	
		0	**+K**	H	E	H	**+I**	H	F	B	G	D	G	C	A	D	A	B	G	H	F	H	F	3	22121336**111**	
			0	**+J**	H	F	H	F	H	G	B	A	D	A	C	G	D	G	B	F	H	**+I**	H	4	22121336**111**	
				0	**+K**	H	E	H	**+I**	H	F	B	G	D	G	C	A	D	A	B	G	H	F	5	22121336**111**	
					0	**+J**	H	F	H	F	H	G	B	A	D	A	C	G	D	G	B	F	H	6	22121336**111**	
						0	**+K**	H	E	H	**+I**	H	F	B	G	D	G	C	A	D	A	B	G	7	22121336**111**	
							0	**+J**	H	F	H	F	H	G	B	A	D	A	C	G	D	G	B	8	22121336**111**	
								0	**+K**	H	E	H	**+I**	H	F	B	G	D	G	C	A	D	A	9	22121336**111**	
									0	**+J**	H	F	H	F	H	G	B	A	D	A	C	G	D	10	22121336**111**	
										0	**+K**	H	E	H	**+I**	H	F	B	G	D	G	C	A	11	22121336**111**	
											0	**+J**	H	F	H	F	H	G	B	A	D	A	C	12	22121336**111**	
												0	**+K**	H	E	H	**+I**	H	F	B	G	D	G	13	22121336**111**	
													0	**+J**	H	F	H	F	H	G	B	A	D	14	22121336**111**	
														0	**+K**	H	E	H	**+I**	H	F	B	G	15	22121336**111**	
															0	**+J**	H	F	H	F	H	G	B	16	22121336**111**	

0	**+K**	H	E	H	**+I**	H	F	**17**	22121336**111**
	0	**+J**	H	F	H	F	H	**18**	22121336**111**
		0	**+K**	H	E	H	**+I**	**19**	22121336**111**
			0	**+J**	H	F	H	**20**	22121336**111**
				0	**+K**	H	E	**21**	22121336**111**
					0	**+J**	H	**22**	22121336**111**
						0	**+K**	**23**	22121336**111**
							0	**24**	22121336**111**

The *binary*(+)*positions* are here recognized on the level of preliminary binary signs. For true recognition of the *binary*(–)*positions* be used the product perfection (2.4 in introduction).

The adjacency matrix ***E*** of ***Tev***:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	*i*	*k*
0	**1**	0	0	0	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	**1**	1
	0	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	0	0	0	**2**	1
		0	**1**	0	o	0	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**3**	1
			0	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**1**	0	**4**	1
				0	**1**	0	0	0	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**5**	1
					0	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**6**	1
						0	**1**	0	0	0	**1**	0	0	0	0	0	0	0	0	0	0	0	0	**7**	1
							0	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**8**	1
								0	**1**	0	0	0	**1**	0	0	0	0	0	0	0	0	0	0	**9**	1
									0	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	**10**	1
										0	**1**	0	0	0	**1**	0	0	0	0	0	0	0	0	**11**	1
											0	**1**	0	0	0	0	0	0	0	0	0	0	0	**12**	1
												0	**1**	0	0	0	**1**	0	0	0	0	0	0	**13**	1
													0	**1**	0	0	0	0	0	0	0	0	0	**14**	1
														0	**1**	0	0	0	**1**	0	0	0	0	**15**	1
															0	**1**	0	0	0	0	0	0	0	**16**	1
																0	**1**	0	0	0	**1**	0	0	**17**	1
																	0	**1**	0	0	0	0	0	**18**	1
																		0	**1**	0	0	0	**1**	**19**	1
																			0	**1**	0	0	0	**20**	1
																				0	**1**	0	0	**21**	1
																					0	**1**	0	**22**	1
																						0	**1**	**23**	1
																							0	**24**	1

The second degree of its adjacency matrix, $\boldsymbol{E}^2$:

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	*k*
1	3	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	1
2	0	3	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	1
3	1	0	3	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1
4	0	1	0	3	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1
5	1	0	1	0	3	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1
6	0	1	0	1	0	3	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1
7	1	0	1	0	1	0	3	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1
8	0	1	0	1	0	1	0	3	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	1
9	0	0	1	0	1	0	1	0	3	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	1
10	0	0	0	1	0	1	0	1	0	3	0	1	0	1	0	1	0	0	0	0	0	0	0	0	1
11	0	0	0	0	1	0	1	0	1	0	3	0	1	1	0	1	0	0	0	0	0	0	0	0	1
12	0	0	0	0	0	1	0	1	0	1	0	3	0	1	0	1	0	1	0	0	0	0	0	0	1
13	0	0	0	0	0	0	1	0	1	0	1	0	3	0	1	0	1	0	1	0	0	0	0	0	1
14	0	0	0	0	0	0	0	1	0	1	0	1	0	3	0	1	0	1	0	1	0	0	0	0	1
15	0	0	0	0	0	0	0	0	1	0	1	0	1	0	3	0	1	0	1	0	1	0	0	0	1
16	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	3	0	1	0	1	0	1	0	0	1
17	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	3	0	1	0	1	0	1	0	1
18	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	3	0	1	0	1	0	1	1
19	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	3	0	1	0	1	0	1
20	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	3	0	1	0	1	1
21	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	3	0	1	0	1
22	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	3	0	1	1
23	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	3	0	1
24	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	1	0	3	1

The second degree matrix no gives perfect information about the binary positions.

We must use the matrices of 6 and 7 degree. The results of matrices $\underline{E^6}$ and E^7 are here connected:

i	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
1	0	258	84	243	75	239	65	191	42	150	33	130	32
2	258	0	248	84	201	75	173	65	139	42	108	33	107
3	84	248	0	258	84	243	75	239	65	191	42	150	33
4	243	84	258	0	248	84	201	75	173	65	139	42	108
5	75	201	84	248	0	258	84	243	75	239	65	191	42
6	239	75	243	84	258	0	248	84	201	75	173	65	139
7	65	173	75	201	84	248	0	258	84	243	75	239	65
8	191	65	239	75	243	84	258	0	248	84	201	75	173
9	42	139	65	173	75	201	84	248	0	258	84	243	75
10	150	42	191	65	239	75	243	84	258	0	248	84	201
11	33	108	42	139	65	173	75	201	84	248	0	258	84
12	130	33	150	42	191	65	239	75	243	84	258	0	248
13	32	107	33	108	42	139	65	173	75	201	84	248	0
14	107	32	130	33	150	42	191	65	239	75	243	84	258
15	33	130	32	107	33	108	42	139	65	173	75	201	84
16	108	33	107	32	130	33	150	42	191	65	239	75	243
17	42	150	33	130	32	107	33	108	42	139	65	173	75
18	139	42	108	33	107	32	130	33	150	42	191	65	239
19	65	191	42	150	33	130	32	107	33	108	42	139	65
20	173	65	139	42	108	33	107	32	130	33	150	42	191
21	75	239	65	191	42	150	33	130	32	107	33	108	42
22	201	75	173	65	139	42	108	33	107	32	130	33	150
23	84	243	75	239	65	191	42	150	33	130	32	107	33
24	248	84	201	75	173	65	139	42	108	33	107	32	130

i	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*k*
1	107	33	108	42	139	65	173	75	201	84	248	1
2	32	130	33	150	42	191	65	239	75	243	84	1
3	130	32	107	33	108	42	139	65	173	75	201	1
4	33	107	32	130	33	150	42	191	65	239	75	1
5	150	33	130	32	107	33	108	42	139	65	173	1
6	42	108	33	107	32	130	33	150	42	191	65	1
7	191	42	150	33	130	32	107	33	108	42	139	1
8	65	139	42	108	33	107	32	130	33	150	42	1
9	239	65	191	42	150	33	130	32	107	33	108	1
10	75	173	65	139	42	108	33	107	32	130	33	1
11	243	75	239	65	191	42	150	33	130	32	107	1
12	84	201	75	173	65	139	42	108	33	107	32	1
13	258	84	243	75	239	65	191	42	150	33	130	1
14	0	248	84	201	75	173	65	139	42	108	33	1
15	248	0	258	84	243	75	239	65	191	42	150	1
16	84	258	0	248	84	201	75	173	65	139	42	1
17	201	84	248	0	258	84	243	75	239	65	191	1
18	75	243	84	258	0	248	84	201	75	173	65	1
19	173	75	201	84	248	0	258	84	243	75	239	1
20	65	239	75	243	84	258	0	248	84	201	75	1
21	139	65	173	75	201	84	248	0	258	84	243	1
22	42	191	65	239	75	243	84	258	0	248	84	1
23	108	42	139	65	173	75	201	84	248	0	258	1
24	33	150	42	191	65	239	75	243	84	258	0	1

We can assert that all the complete identifiers of vertex pairs (ie positions) *are here recognized*. We know that the initial binary signs cannot always be the complete identifiers of vertex pairs, but the clarification is suitable associate these with the results of matrix product E^n of this graph:

1		**2**	**3**	**4**	**5**	**6**			**7**			**8**			**9**	**10**	**11**
-A		***-B***	***-C***	***-D***	***-E***	***-F***			***-G***			***-H***			***+I***	***+J***	***+K***
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
108	107	42	32	33	243	191	201	173	150	139	130	65	75	84	**239**	**248**	**258**
-A1	*-A2*	*-B*	*-C*	*-D*	*-E*	*-F1*	*-F2*	*-F3*	*-G1*	*-G2*	*-G3*	*-H1*	*-H2*	*-H3*	***+I***	***+J***	***+K***
1	*1*	*2*	*1*	*2*	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*2*	*2*	*2*	***1***	***1***	***1***

The number of initial binary signs is 11, the number of perfected binary signs is 18. The last row is here the *frequency vector* for all the rows (vertices) of structure model:

The complete structure model **SM** of ***Tev***:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	*i*	*k*
0	**K**	H3	E	H2	**I**	H1	F1	B	G1	D	G3	C	A2	D	A1	B	G2	H1	F3	H2	F2	H3	**J**	**1**	1
	0	**J**	H3	F2	H2	F3	H1	G2	B	A1	D	A2	C	G3	D	G1	B	F1	H1	**I**	H2	E	H3	**2**	1
		0	**K**	H3	E	H2	**I**	H1	F1	B	G1	D	G3	C	A2	D	A1	B	G2	H1	F3	H2	F2	**3**	1
			0	**J**	H3	F2	H2	F3	H1	G2	B	A1	D	A2	C	G3	D	G1	B	F1	H1	**I**	H2	**4**	1
				0	**K**	H3	E	H2	**I**	H1	F1	B	G1	D	G3	C	A2	D	A1	B	G2	H1	F3	**5**	1
					0	**J**	H3	F2	H2	F3	H1	G2	B	A1	D	A2	C	G3	D	G1	B	F1	H1	**6**	1
						0	**K**	H3	E	H2	**I**	H1	F1	B	G1	D	G3	C	A2	D	A1	B	G2	**7**	1
							0	**J**	H3	F2	H2	F3	H1	G2	B	A1	D	A2	C	G3	D	G1	B	**8**	1
								0	**K**	H3	E	H2	**I**	H1	F1	B	G1	D	G3	C	A2	D	A1	**9**	1
									0	**J**	H3	F2	H2	F3	H1	G2	B	A1	D	A2	C	G3	D	**10**	1
										0	**K**	H3	E	H2	**I**	H1	F1	B	G1	D	G3	C	A2	**11**	1
											0	**J**	H3	F2	H2	F3	H1	G2	B	A1	D	A2	C	**12**	1
												0	**K**	H3	E	H2	**I**	H1	F1	B	G1	D	G3	**13**	1
													0	**J**	H3	F2	H2	F3	H1	G2	B	A1	D	**14**	1
														0	**K**	H3	E	H2	**I**	H1	F1	B	G1	**15**	1
															0	**J**	H3	F2	H2	F3	H1	G2	B	**16**	1
																0	**K**	H3	E	H2	**I**	H1	F1	**17**	1
																	0	**J**	H3	F2	H2	F3	H1	**18**	1
																		0	**K**	H3	E	H2	**I**	**19**	1
																			0	**J**	H3	F2	H2	**20**	1
																				0	**K**	H3	E	**21**	1
																					0	**J**	H3	**22**	1
																						0	**K**	**23**	1
																							0	**24**	1

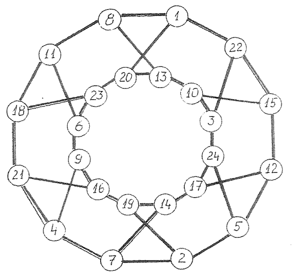
Structural properties:

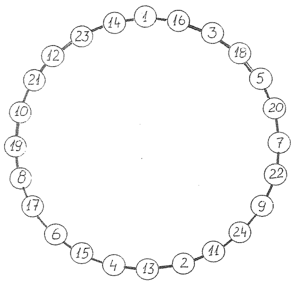
- **a)** Perfected binary sign constitutes a quintuplet ±***d.n.m.e***$^{n}_{ij}$, where the last represents the perfecting.
- **b)** 23x24**:**2 = 276 vertex pairs of ***Tev*** form 18 ***binary positions***, where by 240 "non-edges" be formed 15 binary(–)positions, where the positions with pair signs *–A1, –A2, –C, –E, –F1, –F2, –F3, –G1, –G2, –G3* have 12-elements, and with *–B, –D, –H1, –H2, –H3* have 24 elements.
- **c)** 36 adjacent vertex pairs of ***Tev*** form three binary(+)positions, ***+I***, ***+J*** and ***+K*** that have 12 elements. These are recognized on the level of initial binary signs.
- **d)** The number *N* of binary positions, also position- and adjacent structures is 18, their powers coincide in cases ***Tev*** and ***TevC***, but have reversed signs.
- **e)** Graph ***Tev*** and its complement ***TevC*** divide to 18 ***position-structures***. Position structures by signs *–A1, –A2, –C, –E, –F1, –F2, –F3, –G1, –G2, –G3,* ***I, J,*** and ***K*** have only two pair signs, *–A:–0.2.0* and ***+B:+1.2.1*** and are ***bisymmetric, 2-clique-regular***, and are mutually ***isomorphic***. Position structures by *–B, –D, –H1, –H2* and *–H3* constitutes ***rings*** and ***circlets***.
- **f)** 276 possible ***adjacent graphs*** aggregate to 15 ***adjacent super-structures*** and to three ***adjacent sub-structures***.
- **g)** ***Tev*** is ***bipartite***, in present case parts with even- and odd-numbered vertices.
- **h)** As ***Tev*** is bipartite, but not *bi*-clique, then its complement ***TevC*** consist of two mutually connected *12-cliques* and is thus ***12-clique-regular***. These cliques correspond to parts of ***Tev***.

The ***initial binary signs*** no lose its meaning these show the relationships between vertices, belonging of vertex pairs to (assemblage of) paths or girths with corresponding size etc. It is need for characterizing of the structure. But in our focus are *binary and element (vertex) positions.* Its recognition by help of multiplication the adjacency matrices must be a *mathematical regularity (lawfulness)* for all the no-strongly regular graphs. It could not be ignored. Contra examples do not find. As already mentioned earlier, for strongly regular graphs exist other ways of deep identification.

In the following example is presented a way that is usable also in case of strongly regular graphs.

Example 1.7. The structure of ***Tev*** is recognizable also by pair signs of *sign structure* ***Tev***$_{p=-F}$ (see 2.3 in introduction). Sign structures ***Tev***$_{p=-F}$ and ***Tev***$_{p=-A}$ (these not yet the position structures):





For explanation is suitable to compare the results on the level of *initial binary signs, binary signs of sign structure* and *productive binary signs.*

Initial binary signs ***dnm***$_{ij}$ of ***Tev***, their markings ***p*** and ordering numbers ***n***, ***perfected*** binary signs ***dnm***$_{ij}^{p=F}$ by sign graph ***Tev***$_{p=-F}$, their markings ***p****, ordering numbers ***n**** of binary positions, and ***productive*** binary signs e_{ij}^{6} and e_{ij}^{7} of products E^6 and E^7, where ***6*** and ***7*** is degree of adjacency matrix ***E***.

dnm_{ij}	p	n	$dnm_{ij}^{p=-F}$	p*	n*	e_{ij}^{6}	e_{ij}^{7}
-5.18.23	-A	**1**	-5.10.12	A1	**1**	0	108
			-5.8.8	A2	**2**	0	107
-4.9.10	-B	**2**	-4.7.7	B	**3**	42	0
-4.8.8	-C	**3**	-2.4.4	C	**4**	32	0
-4.7.7	-D	**4**	-2.3.2	D	**5**	33	0
-3.8.9	-E	**5**	-3.10.12	E	**6**	0	243
-3.6.6	-F	**6**	+3.4.4	F1	**7**	0	191
			+5.8.10	F2	**8**	0	201
			+3.4.4	F3	**9**	0	173
-3.4.3	-G	**7**	-3.8.10	G1	**10**	0	150
			-3.6.6	G2	**11**	0	139
			-3.4.3	G3	**12**	0	130
-2.3.2	-H	**8**	-6.20.26	H1	**13**	65	0
			-4.7.7	H2	**14**	75	0
			-2.3.2	H3	**15**	84	0
+5.10.12	**I**	**9**	-3.6.6	**I**	**16**	0	239
+5.12.15	**J**	**10**	-3.4.3	**J**	**17**	0	248
+5.14.18	**K**	**11**	-5.8.8	**K**	**18**	0	258

We see that the same results are obtained by sign structures and products of adjacency matrices coincide.

The positions are essential attributes of structure. The *meaning of the structure (ie its recognition)* consists in its primary attributes – *relationships between elements* and *positions (orbits).* On the other hand *is structure an inseparable attribute of all the really existing objects.*

It has once again demonstrated the importance of structure model in research the graph structure. As well the role of position- and sign-structures in this, and importance of mutual treatment the structure and its complement. All the *hidden sides* are expressed by position- and sign structures.

The importance of position structures lies in the recognition of structural properties, these recognizes the similar parts of various structures. If the structure is divided to parts (bipartite, tripartite etc) or contain components, cliques, girths, etc., then the corresponding attributes appear in position structures in another forms.

The preliminary binary signs their meanings do not lose, these remains characterize belonging the elements and connections to the paths and girths that is needed by treatment of the structure.

2. Partially symmetric structures

Example 2.1. ***Partially symmetric*** structures $\boldsymbol{Pet^{sub}}$ and $\boldsymbol{Pet^{sup}}$ as adjacent structures $\boldsymbol{GS^{adj}}$ of Petersen graph ***Pet*** (example 1.1):

By *removing* at Petersen graph ***Pet*** an edge *i,j*=4,5 is obtained its *adjacent sub-structure* $\boldsymbol{Pet^{sub}}$:

A:-4.10.14; B:-3.6.6; C:-2.3.2; ***D:+4.7.8; E:+4.9.12; F:+4.10.14.***

1	1	1	1	2	2	2	2	3	3		u_i	k	s_i	
2	**6**	**7**	**8**	**1**	**3**	**9**	**10**	**4**	**5**	***i***	*ABC***DEF**		123	***deg***
0	-C	***F***	-C	***E***	***E***	-C	-C	-C	-C	**2**	006**021**	1	120	3
	0	-C	***F***	***E***	-C	***E***	-C	-C	-C	**6**	006**021**	1	120	3
		0	-C	-C	-C	***E***	***E***	-C	-C	**7**	006**021**	1	120	3
			0	-C	***E***	-C	***E***	-C	-C	**8**	006**021**	1	120	3
				0	-C	-C	-C	-B	***D***	**1**	015**120**	2	201	3
					0	-C	-C	***D***	-B	**3**	015**120**	2	201	3
						0	-C	***D***	-B	**9**	015**120**	2	201	3
							0	-B	***D***	**10**	015**120**	2	201	3
								0	-A	**4**	124**200**	3	020	2
									0	**5**	124**200**	3	020	2

By *adding* to Petersen graph ***Pet*** an edge *i,j*=4,6 is obtained its *adjacent super-structure* $\boldsymbol{Pet^{sup}}$:

A:-2.4.4; B:-2.3.2; ***C:+2.3.3; D:+3.4.4; E:+4.10.16.***

1	1	2	3	4	4	4	4	5	5		u_i	k	s_1	s_2	
2	**10**	**7**	**9**	**1**	**3**	**5**	**8**	**4**	**6**	***i***	*AB***CDE**		1234	12345	***deg***
0	-B	***E***	-B	***E***	***E***	-B	-B	-B	-B	**2**	06**003**	1	1020	01020	3
	0	***E***	-B	-B	-B	***E***	***E***	-B	-B	**10**	06**003**	1	1020	01020	3
		0	***E***	-B	-B	-B	-B	-B	-B	**7**	06**003**	2	2100	20100	3
			0	-B	-B	-B	-B	***C***	***C***	**9**	06**201**	3	1002	01002	3
				0	-B	***D***	-B	-A	***D***	**1**	15**021**	4	1011	10011	3
					0	-B	***D***	***D***	-A	**3**	15**021**	4	1011	10011	3
						0	-B	***D***	-A	**5**	15**021**	4	1011	10011	3
							0	-A	***D***	**8**	15**021**	4	1011	10011	3
								0	***C***	**4**	23**220**	5	0121	00121	4
									0	**6**	23**220**	5	0121	00121	4

Structural properties:

- **a)** Exactly these same structures $\boldsymbol{Pet^{sub}}$ and $\boldsymbol{Pet^{sup}}$ are obtainable by operating with an arbitrary edge on Petersen graph.
- **b)** Adjacent sub-structure of Petersen graph has *3 vertex- and 9 binary positions*. Its adjacent super-structure has *5 vertex- and 16 binary positions* and its symmetry value ***SR*** is smaller.
- **c)** From *5-girth regularity* of Petersen graph is in $\boldsymbol{Pet^{sub}}$ remained 14/15 or 93%, but in $\boldsymbol{Pet^{sup}}$ 7/15 or 47%. The first is „more petersenical".
- **d)** *Reverse binary position* that reconstruct the Petersen graph placed in partial model $\boldsymbol{SM_{3.3}}$ of $\boldsymbol{Pet^{sub}}$ by sign –*A*; reconstructing probability ***PF'=1/31***. *Reverse binary position* of $\boldsymbol{Pet^{sup}}$ placed in partial model $\boldsymbol{SM_{5.5}}$ in the form of sign ***C***; reconstructing probability ***PF'=1/16***.
- **e)** Adjacent sub-structure $\boldsymbol{Pet^{sub}}$ is a common adjacent super-structure of 3 initial structures and a common adjacent sub-structure of 6 initial structures. Adjacent super-structure $\boldsymbol{Pet^{sup}}$ is a common adjacent super-structure of 7 initial structures and common adjacent sub-structure of 9 initial structures.
- **f)** Invariants and measures:

G	*|E|*	*K*	*N*	*CL*	*MC*	*DM*	*SVV*	*SV*	SEV^{+}	SE^{+}	*SRV*	*HR*	*SR*
Pet$^{\mathbf{sub}}$	14	3	9	2	5	4	$\mathbf{2^1 4^2}$	*0.5419*	$\mathbf{2^1 4^1 8^1}$	0.6379	$\mathbf{1^1 2^1 4^3 6^1 8^3}$	*0.8939*	***0.4593***
Pet$^{\mathbf{sup}}$	16	5	16	3	5	2	$\mathbf{1^2 2^2 4^1}$	*0.3612*	$\mathbf{1^2 2^3 4^2}$	*0.3437*	$\mathbf{1^3 2^5 4^8}$	*1.1582*	***0.2994***

Example 2.2. Structure model of Boris Weisfeiler's [44, p. 166 (a)] ***partially symmetric*** and ***strongly regular*** graph ***Wei*:**

```
                     A:-2.8.20; B:-2.8.19; C:-2.8.18;
                     D:+2.7.13; E:+2.7.14; F:+2.7.15.
```

```
| 1  1| 2  2| 3  3| 4  4| 5| 6  6| 7  7| 8  8| 9  9|10 10|11|12|13|14 14|15|          u_i     k
|20 24|12 14| 1  2| 9 19| 6|10 16| 8 18| 4  7|11 17|13 15|23| 3|22|21 25| 5|    i  ABCDEF   *
| 0  F| C  C| C  B| F  C| C| B  F| C  E| F  C| E  F| E  C| B| F| F| F  C| F|   20  039039   1
     0| C  C| B  C| C  F| C| F  B| E  C| C  F| F  E| C  E| B| F| F| C  F| F|   24  039039   1
      | 0  F| F  C| C  C| B| F  C| B  F| B  F| E  C| F  E| F| C| F| F  C| E|   12  039039   2
           0| C  F| C  C| B| C  F| F  B| F  B| C  E| E  F| F| C| F| C  F| E|   14  039039   2
            | 0  F| F  C| E| F  C| F  B| F  E| F  C| F  C| C| F| B| C  C| E|    1  039039   3
                 0| C  F| E| C  F| B  F| E  F| C  F| C  F| C| F| B| C  C| E|    2  039039   3
                  | 0  F| E| F  C| F  E| F  C| B  F| F  C| F| B| E| C  B| C|    9  039039   4
                       0| E| C  F| E  F| C  F| F  B| C  F| F| B| E| B  C| C|   19  039039   4
                        | 0| C  C| B  B| B  B| E  E| F  F| F| F| C| F  F| C|    6  066066   5
                           | 0  B| F  E| F  E| B  E| B  E| B| B| C| E  E| C|   10  066066   6
                                0| E  F| E  F| E  B| E  B| B| B| C| E  E| C|   16  066066   6
                                 | 0  C| F  B| F  B| C  C| E| C| B| C  E| E|    8  066066   7
                                      0| B  F| B  F| C  C| E| C| B| E  C| E|   18  066066   7
                                       | 0  C| C  C| B  E| C| E| E| E  B| B|    4  066066   8
                                            0| C  C| E  B| C| E| E| B  E| B|    7  066066   8
                                             | 0  B| E  F| C| B| B| E  C| F|   11  066066   9
                                                  0| F  E| C| B| B| C  E| F|   17  066066   9
                                                   | 0  B| B| C| E| B  F| B|   13  066066  10
                                                        0| B| C| E| F  B| B|   15  066066  10
                                                         | 0| E| D| F  F| E|   23  066147  11
                                                            | 0| E| F  F| D|    3  066147  12
                                                               | 0| B  B| B|   22  093174  13
                                                                  | 0  E| A|   21  147066  14
                                                                       0| A|   25  147066  14
                                                                        | 0|    5  255174  15
```

Structural properties:

- **a)** On the ground of only *six binary signs* is the 25×25 structure matrix decomposed by help *u*- and *s*-vectors to ***15 vertex positions*** and ***115 partial matrices*** $\boldsymbol{W_{ki,kj}}$.
- **b)** 150 "non edges" of ***Wei*** forms ***74 binary*(–)*positions***, where *–A* forms a position with two elements, *–B* forms 33 positions, among these 4 with one element and 29 with two elements, *–C* forms 40 positions, 4 with one, 31 with two and 5 with four elements.
- **c)** 150 edges of ***Wei*** forms ***80 binary*(+)*positions***, where *+**D*** forms 2 positions with one element, *+**E*** 32 positions, among these 4 with one, 27 with two and one with four elements, and *+**F*** forms 46 positions, 6 with one and 40 with two elements.
- **d)** For analyzing the structure of ***Wei*** is suitable use its *sign graphs* $\boldsymbol{Wei_{+2.7.13}}$, $\boldsymbol{Wei_{+2.7.14}}$ and $\boldsymbol{Wei_{+2.7.15}}$.
- **e)** Graph ***Wei*** and its complement ***WeiC*** is *strongly regular, 2-distance-* and *12-degree regular* and *triangular.*
- **f)** Invariants and measures:

G	*|E|*	*k*	N^+	N^-	*P*	*CL*	*G*	*DM*	SEV^+	*SE*	*SVV*	*SV*	*SRV*	*SR*
Wei	150	15	80	74	6	4	3	2	$1^{12}2^{67}4^{1}$	*0.1310*	$1^{5}2^{10}$	*0.1723*	$1^{20}2^{128}4^{6}$	*0.1290*
WeiC	150	15	74	80	6	4	3	2	$1^{8}2^{61}4^{5}$	*0.1494*				

B. Weisfeiler [44] is one of the few who finds that orbits (positions) are essential attributes of graph structure. But he did not reached the binary orbits (positions). He has designed some strongly regular graphs that be grounded on the same binary signs, but are no isomorphic. On structural aspect: these differ from decompositions.

Example 2.3. Structure model of Robertson's ***partially symmetric*** and ***degree regular (4.5)-cage*** [16, p. 272] ***Rob*** and its complement ***RobC*:**

```
                     A:-3.10.12; B:-3.8.9. C:-2.3.2:
                     D:+4.15.24; E:+4.15.25; F:+4.17.31.
```

1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	3	2	3		u_i	k	s_i
13	**14**	**18**	**19**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**15**	**16**	**17**	***i***	*ABCCCDEF*		123
0	B	B	**F**	C2	C1	C2	**E**	C2	C1	C2	**E**	C2	C1	C2	**E**	C2	C2	C2	**13**	0239**0031**	1	130
	0	**F**	B	C1	C2	**E**	C2	C1	C2	**E**	C2	C1	C2	**E**	C2	C2	C2	C2	**14**	0239**0031**	1	130
		0	B	**E**	C2	C1	C2	**E**	C2	C1	C2	**E**	C2	C1	C2	C2	C2	C2	**18**	0239**0031**	1	130
			0	C2	**E**	C2	C1	C2	**E**	C2	C1	C2	**E**	C2	C1	C2	C2	C2	**19**	0239**0031**	1	130
				0	**E**	C1	C1	C2	A	C3	A	C2	C1	C1	**E**	C1	**D**	C1	**1**	2074**1130**	2	121
					0	**E**	C1	C1	C2	A	C3	A	C2	C1	C1	C1	C1	**D**	**2**	2074**1130**	2	121
						0	**E**	C1	C1	C2	A	C3	A	C2	C1	**D**	C1	C1	**3**	2074**1130**	2	121
							0	**E**	C1	C1	C2	A	C3	A	C2	C1	**D**	C1	**4**	2074**1130**	2	121
								0	**E**	C1	C1	C2	A	C3	A	C1	C1	**D**	**5**	2074**1130**	2	121
									0	**E**	C1	C1	C2	A	C3	**D**	C1	C1	**6**	2074**1130**	2	121
										0	**E**	C1	C1	C2	A	C1	**D**	C1	**7**	2074**1130**	2	121
											0	**E**	C1	C1	C2	C1	C1	**D**	**8**	2074**1130**	2	121
												0	**E**	C1	C1	**D**	C1	C1	**9**	2074**1130**	2	121
													0	**E**	C1	C1	**D**	C1	**10**	2074**1130**	2	121
														0	**E**	C1	C1	**D**	**11**	2074**1130**	2	121
															0	**D**	C1	C1	**12**	2074**1130**	2	121
																0	A	A	**15**	2084**0400**	3	040
																	0	A	**16**	2084**0400**	3	040
																		0	**17**	2084**0400**	3	040

A:−2.13.64;
B:+2.11.45; C:+2.11.46; D:+2.12.53; E:+2.12.54; F:+2.12.55.

1	1	1	1	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3		u_i	k	s_i
13	**14**	**18**	**19**	**15**	**16**	**17**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	***i***	*ABCDEF*		123
0	**C**	**C**	−A	**D**	**D**	**D**	**E**	**D**	**E**	−A	**E**	**D**	**E**	−A	**E**	**D**	**E**	−A	**13**	4**02660**	1	239
	0	−A	**C**	**D**	**D**	**D**	**D**	**E**	−A	**E**	**D**	**E**	−A	**E**	**D**	**E**	−A	**E**	**14**	4**02660**	1	239
		0	**C**	**D**	**D**	**D**	−A	**E**	**D**	**E**	−A	**E**	**D**	**E**	−A	**E**	**D**	**E**	**18**	4**02660**	1	239
			0	**D**	**D**	**D**	**E**	−A	**E**	**D**	**E**	−A	**E**	**D**	**E**	−A	**E**	**D**	**19**	4**02660**	1	239
				0	**B**	**B**	**E**	**E**	−A	**E**	**E**	−A	**E**	**E**	−A	**E**	**E**	−A	**15**	4**20480**	2	428
					0	**B**	−A	**E**	**E**	−A	**E**	**E**	−A	**E**	**E**	−A	**E**	**E**	**16**	4**20480**	2	428
						0	**E**	−A	**E**	**E**	−A	**E**	**E**	−A	**E**	**E**	−A	**E**	**17**	4**20480**	2	428
							0	−A	**D**	**D**	**E**	**B**	**F**	**B**	**E**	**D**	**D**	−A	**1**	4**20561**	3	329
								0	−A	**D**	**D**	**E**	**B**	**F**	**B**	**E**	**D**	**D**	**2**	4**20561**	3	329
									0	−A	**D**	**D**	**E**	**B**	**F**	**B**	**E**	**D**	**3**	4**20561**	3	329
										0	−A	**D**	**D**	**E**	**B**	**F**	**B**	**E**	**4**	4**20561**	3	329
											0	−A	**D**	**D**	**E**	**B**	**F**	**B**	**5**	4**20561**	3	329
												0	−A	**D**	**D**	**E**	**B**	**F**	**6**	4**20561**	3	329
													0	−A	**D**	**D**	**E**	**B**	**7**	4**20561**	3	329
														0	−A	**D**	**D**	**E**	**8**	4**20561**	3	329
															0	−A	**D**	**D**	**9**	4**20561**	3	329
																0	−A	**D**	**10**	4**20561**	3	329
																	0	−A	**11**	4**20561**	3	329
																		0	**12**	4**20561**	3	329

Structural properties:

- **a)** Graph ***Rob*** is ***5-girth regular*** with ***4 binary*(+)*positions*** and ***adjacent sub-structures***.
- **b)** Structure models of ***position structure Rob$_{C1}$*** (in $SM_{2.2}$) and its complement ***Rob$_{C1}$C***:

A:−2.4.4; B:−2.3.2;
C:+3.6.7; D:+3.8.10.

A:−2.6.13; B:−2.6.12;
C:+2.5.8; D:+2.6.12; E:+2.6.13.

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	***i***	***k***	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	−A	**C**	**D**	−B	−A	−A	−A	−B	**D**	**C**	−A	**1**	1	0	**D**	−A	−B	**C**	**D**	**E**	**D**	**C**	−B	−A	**D**
	0	−A	**C**	**D**	−B	−A	−A	−A	−B	**D**	**C**	**2**	1		0	**D**	−A	−B	**C**	**D**	**E**	**D**	**C**	−B	−A
		0	−A	**C**	**D**	−B	−A	−A	−A	−B	**D**	**3**	1			0	**D**	−A	−B	**C**	**D**	**E**	**D**	**C**	−B
			0	−A	**C**	**D**	−B	−A	−A	−A	−B	**4**	1				0	**D**	−A	−B	**C**	**D**	**E**	**D**	**C**
				0	−A	**C**	**D**	−B	−A	−A	−A	**5**	1					0	**D**	−A	−B	**C**	**D**	**E**	**D**
					0	−A	**C**	**D**	−B	−A	−A	**6**	1						0	**D**	−A	−B	**C**	**D**	**E**
						0	−A	**C**	**D**	−B	−A	**7**	1							0	**D**	−A	−B	**C**	**D**
							0	−A	**C**	**D**	−B	**8**	1								0	**D**	−A	−B	**C**
								0	−A	**C**	**D**	**9**	1									0	**D**	−A	−B
									0	−A	**C**	**10**	1										0	**D**	−A
										0	−A	**11**	1											0	**D**
											0	**12**	1												0

- **c)** Complement ***RobC*** is ***triangular*** with ***10 binary*(+)*positions*** and ***adjacent sub-structures***. It contain two mutually connected ***6-cliques***, in present case with elements **1,3,5,7,9,11** anf **2,4,6,8,10,12**.

d) Invariants and measures:

G	*|E|*	*k*	N^+	N^-	*P*	*CL*	*G*	*DM*	SEV^+	SE^+	*SRV*	*HR*	*SR*
Rob	38	3	4	10	8	2	5	3	$2^1 12^3$	0.6572	$2^1 3^1 4^1 6^1 12^7 24^3$	0.0685	**0.5215**
RobC	133	3	10	4	6	6	3	2	$3^1 4^1 6^1 12^4 24^3$	0.5653			

Example 2.4. Structure model of graph ***RobB*** that is obtained by adding the *position structure* $\boldsymbol{GS}_{n=-B}$ (ie. edges 13-14, 13-18, 14-19, 18-19) of Robertson's ***partially symmetric*** graph ***Rob*** (example 2.3) to ***Rob*****:**

A:-3.10.13; B:-3.10.12; C:-2.4.4; D:-2.3.2;
E:+2.4.6; F:+3.4.4; G:+3.6.8; H:+4.15.28:

13	14	18	19	15	16	17	1	2	3	4	5	6	7	8	9	10	11	12	*i*	*ABCDDD**EFGH***	*k*	123
0	**E1**	**E1**	**E2**	D1	D1	D1	C	D2	C	**G**	C	D2	C	**G**	C	D2	C	**G**	**13**	006330**3030**	1	303
	0	**E2**	**E1**	D1	D1	D1	D2	C	**G**	C	D2	C	**G**	C	D2	C	**G**	C	**14**	006330**3030**	1	303
		0	**E1**	D1	D1	D1	**G**	C	D2	C	**G**	C	D2	C	**G**	C	D2	C	**18**	006330**3030**	1	303
			0	D1	D1	D1	C	**G**	C	D2	C	**G**	C	D2	C	**G**	C	D2	**19**	006330**3030**	1	303
				0	B	B	D2	D2	**H**	D2	D2	**H**	D2	D2	**H**	D2	D2	**H**	**15**	020480**0004**	2	004
					0	B	**H**	D2	D2	**H**	D2	D2	H	D2	D2	**H**	D2	D2	**16**	020480**0004**	2	004
						0	D2	**H**	D2	D2	**H**	D2	D2	**H**	D2	D2	**H**	D2	**17**	020480**0004**	2	004
							0	**F**	D1	D2	D2	A	D3	A	D2	D2	D1	**F**	**1**	202271**0211**	3	112
								0	**F**	D1	D2	D2	A	D3	A	D2	D2	D1	**2**	202271**0211**	3	112
									0	**F**	D1	D2	D2	A	D3	A	D2	D2	**3**	202271**0211**	3	112
										0	**F**	D1	D2	D2	A	D3	A	D2	**4**	202271**0211**	3	112
											0	**F**	D1	D2	D2	A	D3	A	**5**	202271**0211**	3	112
												0	**F**	D1	D2	D2	A	D3	**6**	202271**0211**	3	112
													0	**F**	D1	D2	D2	A	**7**	202271**0211**	3	112
														0	**F**	D1	D2	D2	**8**	202271**0211**	3	112
															0	**F**	D1	D2	**9**	202271**0211**	3	112
																0	**F**	D1	**10**	202271**0211**	3	112
																	0	**F**	**11**	202271**0211**	3	112
																		0	**12**	202271**0211**	3	112

Structural properties:

- **a)** The *vertex- and binary positions* of ***Rob*** are ***remained*** in ***RobB***, but the number of binary(+)positions is 5.
- **b)** ***RobB*** has a *4-ckique*, is now *not valence regular* and not isomorphic with ***Rob.***

Example 2.5. Structure models of Brinkman's [2, p. 175, Fig. V, 14] ***partially symmetric*** and ***valence regular*** graph ***Bri*** and its complement ***BriC*****:**

A:-3.10.12; B:-3.8.9; C:-3.6.6; D:-2.3.2; ***E:+4.13.19; F:+4.14.19; G:+4.14.21.***

1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3	3		u_i	*k*	s_i
2	**3**	**9**	**10**	**17**	**18**	**21**	**1**	**5**	**6**	**13**	**14**	**19**	**20**	**4**	**7**	**8**	**11**	**12**	**15**	**16**	*i*	*ABC D**EFG***		123
0	-D	-D	**G**	**G**	-D	-D	**G**	**G**	-D	-D	-D	-D	-C	D1	D1	D2	D2	-B	-B	-A	**2**	121 12**004**	1	220
	0	**G**	-D	-D	**G**	-D	**G**	-D	**G**	-D	-D	-C	-D	D1	D2	D1	-B	D2	-A	-B	**3**	121 12**004**	1	220
		0	-D	-D	-D	**G**	-D	**G**	-D	**G**	-C	-D	-D	D2	D1	-B	D1	-A	D2	-B	**9**	121 12**004**	1	220
			0	-D	-D	**G**	-D	-D	**G**	-C	**G**	-D	-D	D2	-B	D1	-A	D1	-B	D2	**10**	121 12**004**	1	220
				0	**G**	-D	-D	-D	-C	**G**	-D	**G**	-D	-B	D2	-A	D1	-B	D1	D2	**17**	121 12**004**	1	220
					0	-D	-D	-C	-D	-D	**G**	-D	**G**	-B	-A	D2	-B	D1	D2	D1	**18**	121 12**004**	1	220
						0	-C	-D	-D	-D	-D	**G**	**G**	-A	-B	-B	D2	D2	D1	D1	**21**	121 12**004**	1	220
							0	D2	D2	D1	D1	-B	-B	-A	**F**	**F**	D2	D2	D1	D1	**1**	121 12**022**	2	202
								0	D1	D2	-B	D1	-B	**F**	-A	D2	**F**	D1	D2	D1	**5**	121 12**022**	2	202
									0	-B	D2	-B	D1	**F**	D2	-A	D1	**F**	D1	D2	**6**	121 12**022**	2	202
										0	-B	D2	D1	D2	**F**	D1	-A	D1	**F**	D2	**13**	121 12**022**	2	202
											0	D1	D2	D2	D1	**F**	D1	-A	D2	**F**	**14**	121 12**022**	2	202
												0	D2	D1	D2	D1	**F**	D2	-A	**F**	**19**	121 12**022**	2	202
													0	D1	D1	D2	D2	**F**	**F**	-A	**20**	121 12**022**	2	202
														0	D2	D2	D1	D1	**E**	**E**	**4**	220 12**220**	3	022
															0	D1	D2	**E**	D1	**E**	**7**	220 12**220**	3	022
																0	**E**	D2	**E**	D1	**8**	220 12**220**	3	022
																	0	**E**	D2	D1	**11**	220 12**220**	3	022
																		0	D1	D2	**12**	220 12**220**	3	022
																			0	D2	**15**	220 12**220**	3	022
																				0	**16**	220 12**220**	3	022

A:−2.15.87; ***B:+2.13.64; C:+2.13.65; D:+2.13.66; E:+2.14.74; F:+2.14.75; G:+2.14.76.***

1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	3	3	3	3		u_i	k	s_i
2	3	9	10	17	18	21	1	5	6	13	14	19	20	4	7	8	11	12	15	16	i	ABCDEFG		123
0	E	E	−A	−A	F	F	−A	−A	F	F	F	F	D	G	G	F	F	C	C	B	2	4121282	1	457
	0	−A	E	F	−A	F	−A	F	−A	F	F	D	F	G	F	G	C	F	B	C	3	4121282	1	457
		0	F	E	F	−A	F	−A	F	−A	D	F	F	F	G	C	G	B	F	C	9	4121282	1	457
			0	F	E	−A	F	F	−A	D	−A	F	F	F	C	G	B	G	C	F	10	4121282	1	457
				0	−A	E	F	F	D	−A	F	−A	F	C	F	B	G	C	G	F	17	4121282	1	457
					0	E	F	D	F	F	−A	F	−A	C	B	F	C	G	F	G	18	4121282	1	457
						0	D	F	F	F	F	−A	−A	B	C	C	F	F	G	G	21	4121282	1	457
							0	F	F	E	E	C	C	B	−A	−A	F	F	G	G	1	4121282	2	565
								0	E	F	C	E	C	−A	B	F	−A	G	F	G	5	4121282	2	565
									0	C	F	C	E	−A	F	B	G	−A	G	F	6	4121282	2	565
										0	C	F	E	F	−A	G	B	G	−A	F	13	4121282	2	565
											0	E	F	F	G	−A	G	B	F	−A	14	4121282	2	565
												0	F	G	F	G	−A	F	B	−A	19	4121282	2	565
													0	G	G	F	F	−A	−A	B	20	4121282	2	565
														0	F	F	E	E	−A	−A	4	4220264	3	754
															0	E	F	−A	E	−A	7	4220264	3	754
																0	−A	F	−A	E	8	4220264	3	754
																	0	−A	F	E	11	4220264	3	754
																		0	E	F	12	4220264	3	754
																			0	F	15	4220264	3	754
																				0	16	4220264	3	754

Common invariants and measures:

G	|E|	k	N^+	N^-	P	CL	G	DM	SEV^+	SE^+	SVV	SV	SRV	SR
Bri	42	3	4	15	7	2	5	3	$7^2 14^2$	0.6443	7^3	0.6391	$7^{12} 14^7 28^1$	0.459
BriC	168	3	15	4	7	7	3	2	$7^{10} 14^5 28^1$	0.4812				

Structural properties:

- **a)** Graph ***Bri*** is ***5-girth regular*** and has ***4 binary(+)positions*** and ***adjacent sub-structures*** with morphism probabilities $PF_1=7/42=1/6$, $PF_2=14/42=1/3$, $PF_3=14/42=1/3$, and $PF_4=7/42=1/6$ correspondingly.
- **b)** Its complement ***BriC*** is ***triangular*** and has ***15 binary(+)positions*** and ***adjacent sub-structures***.
- **c)** A ***7-clique*** of ***BriC*** is expressed in structure model by vertices 1,5,6,13,14,19,20 of second vertex orbit.
- **d)** Data about the number and powers of ***binary orbits*** of ***Bri*** and ***BriC*** contain in symmetry signs ***SRV*** and SEV^+.

3. Structure of real objects

We can assert that a s*ystem, structure* and also *graph* consist of the elements and relationships between the elements, these are "connected sets". How do they relate to each other and how they differ from each other?

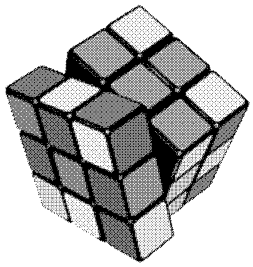
In the system play an important role the *empirical features* of elements and their relationships. Each system has its *function* and *structure*. The structure is a discrete *abstraction* of the system, its "skeleton" where the elements have lost of their empirical meanings but the differences are expressed in the form of different *positions* in the structure. The structure is presentable as a graph, and is associated with the *invariance* and *isomorphism*.

The relations between concepts of the *system, structure, invariance, position* and *graph* are easily and pictorially explainable on the *Rubik's cube*. The function of this cube is known for all. Rubik's cube was studied mainly on playing aspect, but here we interested in its structure. To this end, let's look at a Rubik's Cube and answer to three questions.

<u>Questions:</u>

- **a)** Which positions have the elements of the cube?
- **b)** With layers turning of the cube （different placing are $4{,}3\times10^{19}$）changes its structure or system?
- **c)** How to present a Rubik's cube as a graph

<u>Example 3.1.</u> Rubik's cube as a system that retains the structure (i.e. positions of elements).



<u>Answers:</u>

- **a)** Rubik's cube has in each facet one element in the *middle*, four elements in the *edges* and four elements in the *angles*. Thus, the 6 elements of the cube represent a ***"middle position"***, 24 elements an ***"edge position"*** and 24 elements an ***"angle position"***.
- **b)** With turning the layers of the cube, although be *changed the system*, because the relationships between its empirical properties of the elements (i.e. colors) changes. However, the ***structure does not change***, because the *positions* remains – these are *invariant*. For example, a marked element in *middle* of face will always remain in the middle, and so on.
- **c)** Each element of Rubic's cube has four adjacent elements: an *"upper"*, a *"lower"*, a *"right –hand"*, a *"left-hand"* that are treatable as the ***vertices of a graph.***

Now we need to label the elements and to compile a list of adjacent vertices or adjacency matrix and using it for forming a structure model. This graph is too large and does not make sense to draw it. All the structural information represented in the structure model.

<u>Example 3.2.</u> Processing results: Binary signs and structure model of the Rubik's cube:

A: -8.54.108; B: -7.33.60; C: -7.18.27; D: -6.22.24; E: -6.21.36;
F: -6.20.33; G: -6.19.31: H: -6.15.22; I: -6.12.16; J: -6.12.12;
K: -5.13.20; L: -5.12.17; M: -5.10.13; N: -5.6.5; O: -4.9.12; P: -4. 8.11;
Q: -4.8.10; R: -4.5.4; S: -3.6.7; T: -3.4.3; U: -2.4.4; V: -2.3.2;
W: +2.3.3; X: +3.6.7.

Structure model of **Rubik.GRA** (beginning):

1. “Middle pos”.						**2. “Edge position”**																									
b	r	g	o	y	w	blue				red				green				orange				yellow				**white**					
5	**14**	**23**	**32**	**41**	**50**	**2**	**4**	**6**	**8**	**11**	**13**	**15**	**17**	**20**	**22**	**24**	**26**	**29**	**31**	**33**	**35**	**38**	**40**	**42**	**44**	**47**	**49**	**51**	**53**	***i***	*k*
0	*T*	*D*	*T*	*T*	*T*	**+X**	**+X**	**+X**	**+X**	*Q*	*V*	*R*	*Q*	*N*	*N*	*N*	*N*	*Q*	*R*	*V*	*Q*	*V*	*Q*	*Q*	*R*	*V*	*Q*	*Q*	*R*	**5**	1
	0	*T*	*D*	*T*	*T*	*Q*	*R*	*V*	*Q*	**+X**	**+X**	**+X**	**+X**	*Q*	*V*	*R*	*Q*	*N*	*N*	*N*	*N*	*Q*	*R*	*V*	*Q*	*Q*	*R*	*V*	*Q*	**14**	1
		0	*T*	*T*	*T*	*N*	*N*	*N*	*N*	*Q*	*R*	*V*	*Q*	**+X**	**+X**	**+X**	**+X**	*Q*	*V*	*R*	*Q*	*R*	*Q*	*Q*	*V*	*R*	*Q*	*Q*	*V*	**23**	1
			0	*T*	*T*	*Q*	*V*	*R*	*Q*	*N*	*N*	*N*	*N*	*Q*	*R*	*V*	*Q*	**+X**	**+X**	**+X**	**+X**	*Q*	*V*	*R*	*Q*	*Q*	*V*	*R*	*Q*	**32**	1
				0	*D*	*V*	*Q*	*Q*	*R*	*V*	*Q*	*Q*	*R*	*V*	*Q*	*Q*	*R*	*V*	*Q*	*Q*	*R*	**+X**	**+X**	**+X**	**+X**	*N*	*N*	*N*	*N*	**41**	1
					0	*R*	*Q*	*Q*	*V*	*R*	*Q*	*Q*	*V*	*R*	*Q*	*Q*	*V*	*R*	*Q*	*Q*	*V*	*N*	*N*	*N*	*N*	**+X**	**+X**	**+X**	**+X**	**50**	1
						0	*U*	*U*	*V*	*T*	*S*	*M*	*L*	*R*	*F*	*F*	*J*	*T*	*M*	*S*	*L*	**+X**	*S*	*S*	*T*	*T*	*M*	*M*	*N*	**2**	2
							0	*V*	*U*	*M*	*T*	*N*	*M*	*F*	*J*	*R*	*F*	*S*	*T*	**+X**	*S*	*S*	*T*	*L*	*M*	*S*	*T*	*L*	*M*	**4**	2
								0	*U*	*S*	**+X**	*T*	*S*	*F*	*R*	*J*	*F*	*M*	*N*	*T*	*M*	*S*	*L*	*T*	*M*	*S*	*L*	*T*	*M*	**6**	2
									0	*L*	*S*	*M*	*T*	*J*	*F*	*F*	*R*	*L*	*M*	*S*	*T*	*T*	*M*	*M*	*N*	**+X**	*S*	*S*	*T*	**8**	2
										0	*U*	*U*	*V*	*T*	*S*	*M*	*L*	*R*	*F*	*F*	*J*	*S*	*T*	**+X**	*S*	*M*	*N*	*T*	*M*	**11**	2
											0	*V*	*U*	*M*	*T*	*N*	*M*	*F*	*J*	*R*	*F*	*T*	*M*	*S*	*L*	*T*	*M*	*S*	*L*	**13**	2
												0	*U*	*S*	**+X**	*T*	*S*	*F*	*R*	*J*	*F*	*L*	*M*	*S*	*T*	*L*	*M*	*S*	*T*	**13**	2
													0	*L*	*S*	*M*	*T*	*J*	*F*	*F*	*R*	*M*	*N*	*T*	*M*	*S*	*T*	**+X**	*S*	**17**	2
														0	*U*	*U*	*V*	*T*	*S*	*M*	*L*	*T*	*S*	*S*	**+X**	*N*	*M*	*M*	*T*	**20**	2
															0	*V*	*U*	*M*	*T*	*N*	*M*	*M*	*L*	*T*	*S*	*M*	*L*	*T*	*S*	**22**	2
																0	*U*	*S*	**+X**	*T*	*S*	*M*	*T*	*L*	*S*	*M*	*T*	*L*	*S*	**24**	2
																	0	*L*	*S*	*M*	*T*	*N*	*M*	*M*	*T*	*T*	*S*	*S*	**+X**	**26**	2
																		0	*U*	*U*	*V*	*S*	**+X**	*T*	*S*	*M*	*T*	*N*	*M*	**29**	2
																			0	*V*	*U*	*L*	*S*	*M*	*T*	*L*	*S*	*M*	*T*	**31**	2
																				0	*U*	*T*	*S*	*M*	*L*	*T*	*S*	*M*	*L*	**33**	2
																					0	*M*	*T*	*N*	*M*	*S*	**+X**	*T*	*S*	**35**	2
																						0	*U*	*U*	*V*	*R*	*F*	*F*	*J*	**38**	3
																							0	*V*	*U*	*F*	*R*	*J*	*F*	**40**	2
																								0	*U*	*F*	*J*	*R*	*F*	**42**	2
																									0	*J*	*F*	*F*	*R*	**44**	2
																										0	*U*	*U*	*V*	**47**	2
																											0	*V*	*U*	**49**	2
																												0	*U*	**51**	2
																													0	**53**	2

Structure model of **Rubik.GRA** (ending):

3. “Angle position”																									
blue				red				green				orange				yellow				**white**					
1	**3**	**7**	**9**	**10**	**12**	**16**	**18**	**19**	**21**	**25**	**27**	**28**	**30**	**34**	**36**	**37**	**39**	**43**	**45**	**46**	**48**	**52**	**54**	***i***	***k***
U	*U*	*U*	*U*	*S*	*M*	*S*	*M*	*G*	*G*	*G*	*G*	*M*	*S*	*M*	*S*	*S*	*S*	*M*	*M*	*S*	*S*	*M*	*M*	**5**	1
M	*S*	*M*	*S*	*U*	*U*	*U*	*U*	*S*	*M*	*S*	*M*	*G*	*G*	*G*	*G*	*M*	*S*	*M*	*S*	*M*	*S*	*M*	*S*	**14**	1
G	*G*	*G*	*G*	*M*	*S*	*M*	*S*	*U*	*U*	*U*	*U*	*S*	*M*	*S*	*M*	*M*	*M*	*S*	*S*	*M*	*M*	*S*	*S*	**23**	1
S	*M*	*S*	*M*	*G*	*G*	*G*	*G*	*M*	*S*	*M*	*S*	*U*	*U*	*U*	*U*	*S*	*M*	*S*	*M*	*S*	*M*	*S*	*M*	**32**	1
S	*S*	*M*	*M*	*S*	*S*	*M*	*M*	*S*	*S*	*M*	*M*	*S*	*S*	*M*	*M*	*U*	*U*	*U*	*U*	*G*	*G*	*G*	*G*	**41**	1
M	*M*	*S*	*S*	*M*	*M*	*S*	*S*	*M*	*M*	*S*	*S*	*M*	*M*	*S*	*S*	*G*	*G*	*G*	*G*	*U*	*U*	*U*	*U*	**50**	1
+X	**+X**	*S*	*S*	*V*	*R*	*O*	*H*	*K*	*K*	*B*	*B*	*R*	*V*	*H*	*O*	*U*	*U*	*Q*	*Q*	*Q*	*Q*	*I*	*I*	**2**	2
+X	*S*	**+X**	*S*	*Q*	*I*	*Q*	*I*	*B*	*K*	*B*	*K*	*Q*	*U*	*Q*	*U*	*V*	*O*	*R*	*H*	*V*	*O*	*R*	*H*	**4**	2
S	**+X**	*S*	**+X**	*U*	*Q*	*U*	*Q*	*K*	*B*	*K*	*B*	*I*	*Q*	*I*	*Q*	*O*	*V*	*H*	*R*	*O*	*V*	*H*	*R*	**6**	2
S	*S*	**+X**	**+X**	*O*	*H*	*V*	*R*	*B*	*B*	*K*	*K*	*H*	*O*	*R*	*V*	*Q*	*Q*	*I*	*I*	*U*	*U*	*Q*	*Q*	**8**	2
R	*V*	*H*	*O*	**+X**	**+X**	*S*	*S*	*V*	*R*	*O*	*H*	*K*	*K*	*B*	*B*	*Q*	*U*	*Q*	*U*	*I*	*Q*	*I*	*Q*	**11**	2
Q	*U*	*Q*	*U*	**+X**	*S*	**+X**	*S*	*Q*	*I*	*Q*	*I*	*B*	*K*	*B*	*K*	*R*	*V*	*H*	*O*	*R*	*V*	*H*	*O*	**13**	2
I	*Q*	*I*	*Q*	*S*	**+X**	*S*	**+X**	*U*	*Q*	*U*	*Q*	*K*	*B*	*K*	*B*	*H*	*O*	*R*	*V*	*H*	*O*	*R*	*V*	**15**	2
H	*O*	*R*	*V*	*S*	*S*	**+X**	**+X**	*O*	*H*	*V*	*R*	*B*	*B*	*K*	*K*	*I*	*Q*	*I*	*Q*	*Q*	*U*	*Q*	*U*	**17**	2
K	*K*	*B*	*B*	*R*	*V*	*H*	***O***	**+X**	**+X**	*S*	*S*	*V*	*R*	*O*	*H*	*Q*	*Q*	*U*	*U*	*I*	*I*	*Q*	*Q*	**20**	2
B	*K*	*B*	*K*	*Q*	*U*	*Q*	*U*	**+X**	*S*	**+X**	*S*	*Q*	*I*	*Q*	*I*	*H*	*R*	*O*	*V*	*H*	*R*	*O*	*V*	**22**	2
K	*B*	*K*	*B*	*I*	*Q*	*I*	*Q*	*S*	**+X**	*S*	**+X**	*U*	*Q*	*U*	*Q*	*R*	*H*	*V*	*O*	*R*	*H*	*V*	*O*	**24**	2
B	*B*	*K*	*K*	*H*	*O*	*R*	*V*	*S*	*S*	**+X**	**+X**	*O*	*H*	*V*	*R*	*I*	*I*	*Q*	*Q*	*Q*	*Q*	*U*	*U*	**26**	2
V	*R*	*O*	*H*	*K*	*K*	*B*	*B*	*R*	*V*	*H*	*O*	**+X**	**+X**	*S*	*S*	*U*	*Q*	*U*	*Q*	*Q*	*I*	*Q*	*I*	**29**	2
Q	*I*	*Q*	*I*	*B*	*K*	*B*	*K*	*Q*	*U*	*Q*	*U*	**+X**	*S*	**+X**	*S*	*O*	*H*	*V*	*R*	*O*	*H*	*V*	*R*	**31**	2
U	*Q*	*U*	*Q*	*K*	*B*	*K*	*B*	*I*	*Q*	*I*	*Q*	*S*	**+X**	*S*	**+X**	*V*	*R*	*O*	*H*	*V*	*R*	*O*	*H*	**33**	2
O	*H*	*V*	*R*	*B*	*B*	*K*	*K*	*H*	*O*	*R*	*V*	*S*	*S*	**+X**	**+X**	*Q*	*I*	*Q*	*I*	*U*	*Q*	*U*	*Q*	**35**	2
U	*U*	*Q*	*Q*	*V*	*O*	*R*	*H*	*Q*	*Q*	*I*	*I*	*O*	*V*	*H*	*R*	**+X**	**+X**	*S*	*S*	*K*	*K*	*B*	*B*	**38**	2
V	*O*	*R*	*H*	*Q*	*Q*	*I*	*I*	*O*	*V*	*H*	*R*	*U*	*U*	*Q*	*Q*	**+X**	*S*	**+X**	*S*	*K*	*B*	*K*	*B*	**40**	2
O	*V*	*H*	*R*	*U*	*U*	*Q*	*Q*	*V*	*O*	*R*	*H*	*Q*	*Q*	*I*	*I*	*S*	**+X**	*S*	**+X**	*B*	*K*	*B*	*K*	**42**	2
Q	*Q*	*I*	*I*	*O*	*V*	*H*	*R*	*U*	*U*	*Q*	*Q*	*V*	*O*	*R*	*H*	*S*	*S*	**+X**	**+X**	*B*	*B*	*K*	*K*	**44**	2
Q	*Q*	*U*	*U*	*R*	*H*	*V*	*O*	*I*	*I*	*Q*	*Q*	*H*	*R*	*O*	*V*	*K*	*K*	*B*	*B*	**+X**	**+X**	*S*	*S*	**47**	2
R	*H*	*V*	*O*	*I*	*I*	*Q*	*Q*	*H*	*R*	*O*	*V*	*Q*	*Q*	*U*	*U*	*K*	*B*	*K*	*B*	**+X**	*S*	**+X**	*S*	**49**	2
H	*R*	*O*	*V*	*Q*	*Q*	*U*	*U*	*R*	*H*	*V*	*O*	*I*	*I*	*Q*	*Q*	*B*	*K*	*B*	*K*	*S*	**+X**	*S*	**+X**	**51**	2
I	*I*	*Q*	*Q*	*H*	*R*	*O*	*V*	*Q*	*Q*	*U*	*U*	*R*	*H*	*V*	*O*	*B*	*B*	*K*	*K*	*S*	*S*	**+X**	**+X**	**53**	2
0	*V*	*V*	*O*	*T*	*N*	*L*	*C*	*E*	*P*	*A*	*E*	*T*	**+W**	*L*	*S*	**+W**	*S*	*T*	*L*	*T*	*L*	*N*	*C*	**1**	3
	0	*O*	*V*	**+W**	*T*	*S*	*L*	*P*	*E*	*E*	*A*	*N*	*T*	*C*	*L*	*S*	**+W**	*L*	*T*	*L*	*T*	*C*	*N*	**3**	3
		0	*V*	*L*	*C*	*T*	*N*	*A*	*E*	*E*	*P*	*L*	*S*	*T*	**+W**	*T*	*L*	*N*	*C*	**+W**	*S*	*T*	*L*	**7**	3
			0	*S*	*L*	**+W**	*T*	*E*	*A*	*P*	*E*	*C*	*L*	*N*	*T*	*L*	*T*	*C*	*N*	*S*	**+W**	*L*	*T*	**9**	3
				0	*V*	*V*	*O*	*T*	*N*	*L*	*C*	*E*	*P*	*A*	*E*	*T*	**+W**	*L*	*S*	*N*	*T*	*C*	*L*	**10**	3

```
  0   O   V  +W   T   S   L   P   E   E   A   L   S   T  +W   C   L   N   T | 12  3
      0   V   L   C   T   N   A   E   E   P   N   T   C   L   T  +W   L   S | 16  3
          0   S   L  +W   T   E   A   P   E   C   L   N   T   L   S   T  +W | 18  3
              0   V   V   O   T   N   L   C   L   T   S  +W   C   N   L   T | 19  3
                  0   O   V  +W   T   S   L   T   L  +W   S   N   C   T   L | 21  3
                      0   V   L   C   T   N   C   N   L   T   L   T   S  +W | 25  3
                          0   S   L  +W   T   N   C   T   L   T   L  +W   S | 27  3
                              0   V   V   O   S   L  +W   T   L   C   T   N | 28  3
                                  0   O   V  +W   T   S   L   T   N   L   C | 30  3
                                      0   V   L   C   T   N   S   L  +W   T | 34  3
                                          0   T   N   L   C  +W   T   S   L | 36  3
                                              0   V   V   O   P   E   E   A | 37  3
                                                  0   O   V   E   P   A   E | 39  3
                                                      0   V   E   A   P   E | 43  3
                                                          0   A   E   E   P | 45  3
                                                              0   V   V   O | 46  3
                                                                  0   O   V | 48  3
                                                                      0   V | 52  3
                                                                          0 | 54  3
```

The elements of cube are labeled in this case by the facets, but the algorithm divides these on the basis of frequency- $\boldsymbol{u_i}$ and position $\boldsymbol{s_i}$ vectors into positions $\boldsymbol{k}$. The binary signs in the model characterize the relationships between the elements, where in the present case exists 24 types of relationships. The frequency vector represents the relationships of an element $\boldsymbol{i}$ with other elements. The position vector represents the relationships of an element $\boldsymbol{i}$ with the elements in the same and other positions $\boldsymbol{k}$.

Example 3.3. Frequency- and position vectors of elements of Rubik's cube and corresponding positions:

Frequency vectors																								Position vectors					
A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*	*N*	*O*	*P*	*Q*	*R*	*S*	*T*	*U*	*V*	***W***	***X***	***k***	**1**	**2**	**3**	**Positions**	**No**
0	0	0	1	0	0	4	0	0	0	0	0	8	4	0	0	8	4	8	4	4	4	**0**	**4**	**1**	**0**	**4**	**0**	**"Middle position"**	**6**
0	2	0	0	0	2	0	2	2	1	2	2	4	2	2	0	6	4	6	4	4	4	**0**	**4**	**2**	**1**	**1**	**2**	**"Edge position"**	**24**
1	2	2	0	2	0	1	2	2	0	2	4	2	2	3	1	4	2	6	4	3	4	**2**	**2**	**3**	**0**	**2**	**2**	**"Angle position"**	**24**

Therefore, structure model is the *complete invariant* of Rubil's cube where: 1) the elements 5, 14, 23, 32, 41, 50 are on the "middle position", 2) the elements 2, 4, 6, 8, 11, 13, 15, 17, 20, 22, 24, 26, 29, 31, 33, 35, 38, 40, 42, 44, 47, 49, 51, 53 on the "edge position", 3) and the elements 1, 3, 7, 9, 10, 12, 16, 18, 19, 21, 25, 27, 28, 30, 34, 36, 37, 39, 43, 45, 46, 48, 52, 54 on the "angle position".

With turning a layer exchanged the place (replaced) some labels of elements, i. e. *changes the system, but the structure (relationships between the elements) remain invariable.* Essentially, deal with replacing of rows and columns (or changing of labeling), which does not really make sense, since we obtain the *equivalent structural models*, i. e. *isomorphic graphs* of Rubic's cube.

The structure of Rubik's graph has also 41 *binary positions* (positions that consist in corresponding vertex pairs). These are:

1. In segment 1.1 exist *two* binary positions, with vertex pairs ***D*** and ***T***;
2. In segment 1.2 exist *five* binary positions, with vertex pairs ***N, Q, R, V*** and ***+X***;
3. In segment 1.3 exist *four* binary positions, with vertex pairs ***G, M, S*** and ***U***;
4. In segment 2.2 exist *nine* binary positions, with vertex pairs ***F, J, M, N, R, S, T, U*** and ***V***;
5. In segment 2.3 exist *ten* binary positions, with vertex pairs ***B, H, K, O, Q, R, S, U, V*** and ***+X***;
6. In segment 3.3 exist *eleven* binary positions, with vertex pairs ***A, C, E, F, L, N, O, S, T, V*** and ***+W***.

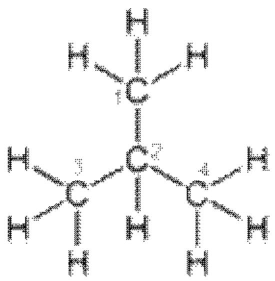
Here we have discussed about the structure of Rubik's cube, but not about the playing with it. As the positions of elements of this cube are expressed by binary signs and the colours of elements are known, then may be possible to construct *a version of playing on the basis of structure's model* **SM.** In principle can be constructed various plays on the ground of this model.

*

Structure model of the ***chemical compound*** is a detailed submission of the classical structural formula, i.e. of a graph that represents this formula.

This is a so-called *systemic approach* to the study of chemical compounds where different chemical elements (atoms) as a rule, are divided into different *positions* as *subsystems*. In case of more complex compounds, however, may also the same elements (atoms) belong to different positions (for example, ethanol, butane, propane, etc.). The main idea of using the same models consists in treatment of the whole on the basis of positions and the relationships between them. Structural models open up the possibility for additional investigation of chemical compounds. The structural models of some polymers and organic matters tend to be very large. Here is limited with moderates.

<u>Example 3.4.</u> Structural formula of *isobutan* C_4H_{10}, its binary signs and structure model:

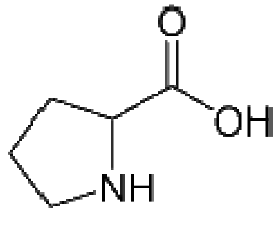


A:-4.5.4; B:-3.4.3; C:-2.3.2;
D:+1.2.1.

2	1	3	4	11	5	6	7	8	9	10	12	13	14		*i*	u_i		s_i
C	**C**	**C**	**C**	**H**	**H**	**H**	**H**	**H**	**H**	**H**	**H**	**H**	**H**	***a***		*ABC**D***	***k***	1234
0	***D***	***D***	***D***	***D***	*-C*	*-C*	*-C*	*-C*	*-C*	*-C*	*-C*	*-C*	*-C*	**C**	2	*009**4***	*1*	0**31**0
	0	*-C*	*-C*	*-C*	***D***	***D***	***D***	*-B*	*-B*	*-B*	*-B*	*-B*	*-B*	**C**	1	*063**4***	*2*	**1**00**3**
		0	*-C*	*-C*	*-B*	*-B*	*-B*	***D***	***D***	***D***	*-B*	*-B*	*-B*	**C**	3	*063**4***	*2*	**1**00**3**
			0	*-C*	*-B*	*-B*	*-B*	*-B*	*-B*	*-B*	***D***	***D***	***D***	**C**	4	*063**4***	*2*	**1**00**3**
				0	*-B*	*-B*	*-B*	*-B*	*-B*	*-B*	*-B*	*-B*	*-B*	**H**	11	*093**1***	*3*	**1**000
					0	*-C*	*-C*	*-A*	*-A*	*-A*	*-A*	*-A*	*-A*	**H**	5	*633**1***	*4*	0**1**00
						0	*-C*	*-A*	*-A*	*-A*	*-A*	*-A*	*-A*	**H**	6	*633**1***	*4*	0**1**00
							0	*-A*	*-A*	*-A*	*-A*	*-A*	*-A*	**H**	7	*633**1***	*4*	0**1**00
								0	*-C*	*-C*	*-A*	*-A*	*-A*	**H**	8	*633**1***	*4*	0**1**00
									0	*-C*	*-A*	*-A*	*-A*	**H**	9	*633**1***	*4*	0**1**00
										0	*-A*	*-A*	*-A*	**H**	10	*633**1***	*4*	0**1**00
											0	*-C*	*-C*	**H**	12	*633**1***	*4*	0**1**00
												0	*-C*	**H**	13	*633**1***	*4*	0**1**00
													0	**H**	14	*633**1***	*4*	0**1**00

Explanation: Decomposition of the elements C and H to four positions should not cause questions.

<u>Example 3.5.</u> Structural formula, binary signs and structure model of the amino acid *proline* $C_5H_9NO_2$:



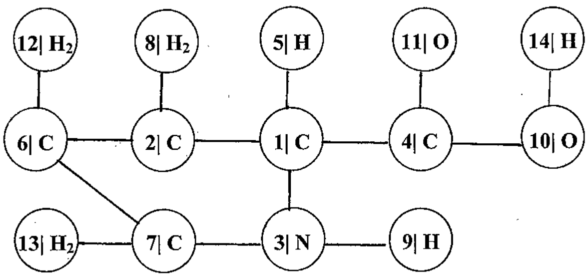
A:-6.7.6; B:-5.6.5; C:-4.5.4; D:-3.4.3; E:-2.3.2;
F:+1.2.1; G:+4.5.5.

4	3	15	5	10	2	1	8	9	12	17	16	6	7	13	14	11		i	u_i	
C	**C**	**N**	**C**	**H**	**C**	**C**	**H**	**H**	**H**	**O**	**O**	**H**	**H**	**H**	**H**	**H**	***a***		*ABCDE**FG***	***k***
0	***G***	***G***	***F***	***F***	*-E*	*-E*	*-E*	*-E*	*-E*	*-E*	*-E*	*-D*	*-D*	*-D*	*-D*	*-D*	**C**	4	*00057**22***	*1*
	0	*-E*	*-E*	*-E*	***G***	*-E*	***F***	***F***	*-D*	*-D*	*-D*	*-E*	*-E*	*-D*	*-D*	*-C*	**C**	3	*00156**22***	*2*
		0	*-E*	*-E*	*-E*	***G***	*-D*	*-D*	***F***	*-D*	*-D*	*-D*	*-D*	*-E*	*-E*	*-C*	**N**	15	*00166**12***	*3*
			0	*-E*	*-D*	*-D*	*-D*	*-D*	*-D*	***F***	***F***	*-C*	*-C*	*-C*	*-C*	*-E*	**C**	5	*00454**30***	*4*
				0	*-D*	*-D*	*-D*	*-D*	*-D*	*-D*	*-D*	*-C*	*-C*	*-C*	*-C*	*-C*	**H**	10	*00573**10***	*5*
					0	***G***	*-E*	*-E*	*-D*	*-C*	*-C*	***F***	***F***	*-E*	*-E*	*-B*	**C**	2	*01236**22***	*6*
						0	*-D*	*-D*	*-E*	*-C*	*-C*	*-E*	*-E*	***F***	***F***	*-B*	**C**	1	*01245**22***	*7*
							0	*E*	*-C*	*-C*	*-C*	*-D*	*-D*	*-C*	*-C*	*-B*	**H**	8	*01563**10***	*8*
								0	*-C*	*-C*	*-C*	*-D*	*-D*	*-C*	*-C*	*-B*	**H**	9	*01563**10***	*8*
									0	*-C*	*-C*	*-C*	*-C*	*-D*	*-D*	*-B*	**H**	12	*01662**10***	*9*
										0	*-E*	*-B*	*-B*	*-B*	*-B*	***F***	**O**	17	*04532**20***	*10*
											0	*-B*	*-B*	*-B*	*-B*	*-D*	**O**	16	*04542**10***	*11*
												0	*-E*	*-D*	*-D*	*-A*	**H**	6	*12363**10***	*12*
													0	*-D*	*-D*	*-A*	**H**	7	*12363**10***	*12*
														0	*-E*	*-A*	**H**	13	*12453**10***	*13*
															0	*-A*	**H**	14	*12453**10***	*13*
																0	**H**	11	*45321**10***	*14*

			s_i
a	***i***	***k***	*12345678901234*
C	4	*1*	0**1111**000000000
C	3	*2*	**1**00001**0**2000000
N	15	*3*	**1**000001**0**100000
C	5	*4*	**1**000000000**11**000
H	10	*5*	**1**0000000000000
C	2	*6*	0**1**0000**1**0000**2**00
C	1	*7*	00**1**00**1**000000**2**0
H	8	*8*	0**1**000000000000
H	9	*8*	0**1**000000000000
H	12	*9*	00**1**00000000000
O	17	*10*	000**1**000000000**1**
O	16	*11*	000**1**0000000000
H	6	*12*	00000**1**00000000
H	7	*12*	00000**1**00000000
H	13	*13*	000000**1**0000000
H	14	*13*	000000**1**0000000
H	11	*14*	0000000000**1**0000

Structure model of proline also provides all the relationship between the elements. Its 17 elements are concentrated in 14 different positions. Presented separately position-vectors $\boldsymbol{s_i}$ constitutes an adjacent matrix of positions, which enable to compose corresponding "position's graph".

<u>Example 3.6.</u> The *position's graph* of proline:



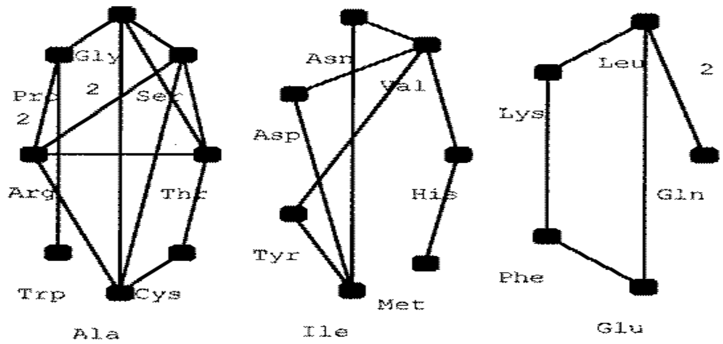
The structure model of this position graph ("positions model") we here no represents, we note only that this has 10 positions, in which joined former positions (2, 3), (6, 7), (8, 9) and (12, 13). The structure model of chemical compound opens for chemist unfamiliar structural side, but this side does not advisable to ignore because the existence of structure is real. To this end, all of this is presented here.

Exists also such a thing as a “chemical graph theory”, which can be regarded as the mainstay of chemical compounds in the field of work by Arthur Cayley in 1874 (although if the term “graph” was not yet used). The end of the last century, thousands of articles on the subject, and in 1980 published a two-volume monograph of Nenad Trinaisti on *Chemical Graph Theory*. Proponents of this theory argue that it is giving valuable information about chemical phenomena, however, to the opponents seems it reasonable only in exceptional cases. Recommend to support the first. Moreover, the structure model is something perfect than a graph.

*

The ***genetic code*** in biology describes how genes that are composed of DNA are translated into proteins composed amino acids. The American bioinformatics William Seffens seems that genetic codes can be represented as *graphs* where the elements are *amino acids.* In his article, he justifies this view-point [18]. Here we limited with the treatment of graphs and structural model of the genetic code.

Example 3.7. The graph with three components of *Standard genetic code* (ID=1), its binary signs and structure model:



A: -4.7.9; B:-4.7.8; C:-3.6.8; D:-3.6.7; E:-3.5.5; F:-3.4.3;
G:-2.6.10; H:-2.5.6; I:-2.4.4; J:-2.3.2; K:-u.2.0;
L:+1.2.1; M:+2.3.3; N:+2.4.5; O:+3.4.4; P:+3.5.6; Q:+3.6.10.

L	**E**	**K**	**V**	**R**	**G**	**N**	**D**	**Y**	**Q**	**F**	**S**	**A**	**T**	**H**	**P**	**M**	**I**	**W**	**C**		***a***		
11	7	12	20	2	8	3	4	19	6	14	16	1	17	9	15	13	10	18	5	***i***		***deg***	***k***
0	***O***	***O***	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	***L***	*-I*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	11	**Leu**	*3*	*1*
	0	*-I*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-J*	***O***	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	7	**Glu**	*2*	*2*
		0	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-J*	***O***	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	12	**Lys**	*2*	*2*
			0	*-K*	*-K*	***P***	***P***	***P***	*-K*	*-K*	*-K*	*-K*	*-K*	***L***	*-K*	*-J*	*-H*	*-K*	*-K*	20	**Val**	*4*	*3*
				0	*-G*	*-K*	*-K*	*-K*	*-K*	*-K*	***N***	***M***	***M***	*-K*	***Q***	*-K*	*-K*	*-J*	*-I*	2	**Arg**	*4*	*4*
					0	*-K*	*-K*	*-K*	*-K*	*-K*	***N***	***M***	***M***	*-K*	***Q***	*-K*	*-K*	*-J*	*-I*	8	**Gly**	*4*	*4*
						0	*-I*	*-I*	*-K*	*-K*	*-K*	*-K*	*-K*	*-J*	*-K*	*-F*	***P***	*-K*	*-K*	3	**Asn**	*2*	*5*
							0	*-I*	*-K*	*-K*	*-K*	*-K*	*-K*	*-J*	*-K*	*-F*	***P***	*-K*	*-K*	4	**Asp**	*2*	*5*
								0	*-K*	*-K*	*-K*	*-K*	*-K*	*-J*	*-K*	*-F*	**P**	*-K*	*-K*	19	**Tyr**	*2*	*5*
									0	*-E*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	6	**Gln**	*1*	*6*
										0	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	*-K*	14	**Phe**	*2*	*7*
											0	***N***	***N***	*-K*	*-I*	*-K*	*-K*	*-E*	*-I*	16	**Ser**	*4*	*8*
												0	*-G*	*-K*	*-I*	*-K*	*-K*	*-E*	***Q***	1	**Ala**	*4*	*9*
													0	*-K*	*-I*	*-K*	*-K*	*-E*	***Q***	17	**Thr**	*4*	*9*
														0	*-K*	***L***	*-D*	*-K*	*-K*	9	**His**	*2*	*10*
															0	*-K*	*-K*	***L***	*-C*	15	**Pro**	*3*	*11*
																0	*-B*	*-K*	*-K*	13	**Met**	*1*	*12*
																	0	*-K*	*-K*	10	**Ile**	*3*	*13*
																		0	*-A*	18	**Trp**	*1*	*14*
																			0	5	**Cys**	*2*	*15*

i	a	u_i ABCDEFGHIJ	K	**LMNOPQ**	k	s_i 123456789012345
11	**Leu**	0000000010	15	**100200**	1	02000**1**000000000
7	**Glu**	0000000011	15	**000200**	2	**1**00000**1**00000000
12	**Lys**	0000000011	15	**000200**	2	**1**00000**1**00000000
20	**Val**	0000000101	13	**100030**	3	0000**3**0000**1**00000
2	**Arg**	0000001011	12	**021001**	4	0000000**12**0**1**0000
8	**Gly**	0000001011	12	**021001**	4	0000000**12**0**1**0000
3	**Asn**	0000010021	13	**000020**	5	00**1**00000000**0**100
4	**Asp**	0000010021	13	**000020**	5	00**1**000000000**1**00
19	**Tyr**	0000010021	13	**000020**	5	00**1**000000000**1**00
6	**Gln**	0000100002	15	**100000**	6	**1**00000000000000
14	**Phe**	0000100010	15	**000200**	7	0**2**0000000000000
16	**Ser**	0000100020	12	**004000**	8	000**2**0000**2**000000
1	**Ala**	0000101010	12	**021001**	9	000**2**000**1**000000**1**
17	**Thr**	0000101010	12	**021001**	9	000**2**000**1**000000**1**
9	**His**	0001000003	13	**200000**	10	00**1**00000000**1**000
15	**Pro**	0010000030	12	**100002**	11	000**2**000000000**1**0
13	**Met**	0100030001	13	**100000**	12	000000000**1**00000
10	**Ile**	0101000100	13	**000030**	13	0000**3**0000000000
18	**Trp**	1000300002	12	**100000**	14	0000000000**1**0000
5	**Cys**	1010000030	12	**000002**	15	00000000**2**000000

Three components are by Seffens represented as a common graph, because in case of alternative genetic codes exists relationships between the components. The numbers by relations indicate the number of edges (multigraph's existence). The girths exist in standard genetic code with a length of 3 and 4. Disconnections with the other components represent binary sign *K:–u.2.0.* Twenty amino acids form the fifteen positions. We can see that the common positions ***k*** in the genetic code have the following amino acids

k=2: *glutamic acid* (Glu) ja *lysine* (Lys); ***k=4:*** *arginine* (Arg) ja *glycine* (Gly);
k=5: *aspargine* (Asn), *aspartic acid* (Asp) ja *tyrosine* (Tyr); ***k=9:*** *alahine* (Ala) ja *threonine* (Thr).

If accepted the positions in genetic code, then should also be accept the relationships between positions (position vectors s_i), which constitutes the adjacent matrix of positions. The corresponding graph to present here does not make sense, but the structural model can be set up. Existing there double and triple connections can be ignore, because these characterize only the number of amino acids that having a common position.

Example 3.8. Binary signs and structure model of *position's relationships* of Standard genetic code:

A:-4.5.4; B:-3.4.3; C:-2.3.2; D:+u.2.0; ***E:+1.2.1; F:+2.3.3.***

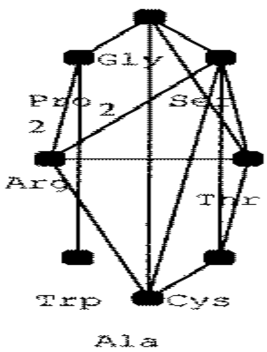
1	2	4	3	9	5	10	6	7	8	11	12	13	15	14	**k**	u_i ABC	D	**EF**	**k***
0	**E**	-D	-D	-D	-D	-D	**E**	-C	-D	-D	-D	-D	-D	-D	1	001	11	**20**	1
	0	-D	-D	-D	-D	-D	-C	**E**	-D	-D	-D	-D	-D	-D	2	001	11	**20**	1
		0	-D	**F**	-D	-D	-D	-D	**F**	**E**	-D	-D	-C	-C	4	002	9	**12**	2
			0	-D	**E**	**E**	-D	-D	-D	-D	-C	-C	-D	-D	3	002	10	**20**	3
				0	-D	-D	-D	-D	**F**	-C	-D	-D	**E**	-B	9	011	9	**12**	4
					0	-C	-D	-D	-D	-D	-B	**E**	-D	-D	5	011	10	**20**	5
						0	-D	-D	-D	-D	**E**	-B	-D	-D	10	011	10	**20**	5
							0	-B	-D	-D	-D	-D	-D	-D	6	011	11	**10**	6
								0	-D	-D	-D	-D	-D	-D	7	011	11	**10**	6
									0	-C	-D	-D	-C	-B	8	012	9	**02**	7
										0	-D	-D	-B	**E**	11	012	9	**20**	8
											0	-A	-D	-D	12	111	10	**10**	9
												0	-D	-D	13	111	10	**10**	9
													0	-A	15	112	9	**10**	10
														0	14	121	9	**10**	11

k	s_i 12345678901	k*	a	*
1	10000100000	1	Leu	
2	10000100000	1	Glu, Lys	*
4	00010011000	2	Arg, Gly	*
3	00002000000	3	Val	*
9	01000010010	4	Ala, Thr	*
5	00100000100	5	Asn, Asp,	*
10	00100000100	5	His, Tyr	*
6	10000000000	6	Gln	*
7	10000000000	6	Phe	*
8	01010000000	7	Ser	*
11	01000000001	8	Pro	*
12	00001000000	9	Met	*
13	00001000000	9	Ile	*
15	00010000000	10	Cys	*
14	00000001000	11	Trp	*

In the structure model of *position's relationships* are some previous positions ***k*** merged into new positions ***k****, the number of previous positions was 15, now 11. Also here represent the position vectors a new adjacent matrix, on which it could continue to operate. However we limited, because the genetic significances of the obtained results are not covered here.

Alternative genetic codes differ from standard code a greater or lesser extent. Different components of the codes can be isomorphic. For example, the second and third component of Euplotid Nuclear code (ID=10) is isomorphic with the corresponding components of Standard genetic code, etc. The differences expressed as a few different loop, a new relationship (edge) in component or between components.

Example 3.9. First component of *Euplotid Nuclear code* (ID=10), its binary signs and structure model:



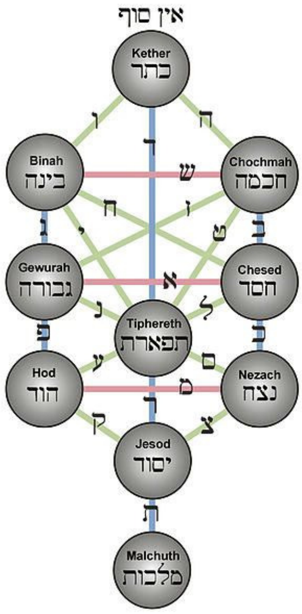
A:-4.8.14; B:-3.7.13; C:-3.5.5; D:-2.6.11; E:-2.6.10; F:-2.5.8; G:-2.4.4; H:-2.3.2;
I:+1.2.1; J:+2.3.3; K:+2.4.5; L:+2.5.7; M:+3.6.10.

G	R	S	T	A	P	W	C		a	u_i		s_i
8	2	16	17	1	15	18	5	i		ABCDEFGHIJKLM	k	123456
0	-E	K	J	J	M	-H	-F	8	Gly	0000110102101	1	012100
	0	K	J	J	M	-H	-F	2	Arg	0000110102101	1	012100
		0	L	L	-G	-C	K	16	Ser	0010001000320	2	202001
			0	-D	-G	-C	J	17	Thr	0011001003010	3	210001
				0	-G	-C	J	1	Ala	0011001003010	3	210001
					0	I	-B	15	Pro	0100003010002	4	200010
						0	-A	18	Trp	1030000210000	5	000100
							0	5	Cys	1100020002100	6	012000

Euplotid Nuclear code is an *adjacent superstructure* of Standard code, it differs only by addition of a relation (connection) between Ser and Cys to the first component. Changes the structure, but the positions will be retained. W. Seffens has treated 15 genetic codes, which on the structural aspect forms a "space of genetic codes".

*

Example 3,10. Partially symmetric graph of Kabalahh ***Kab*** and its structure model**:**



Where: Kether – *Crown*; Binach – *Understanding*; Chochmah – *Wisdom*; Gewurah – *Severity*; Chesed – *Mercy*; Tiphereth – *Beauty*; Hod – *Splendor*; Nezah – *Victory*; Jesod – *Foundation*; Malchuth – *Kingdom*.

```
        A: -3.5.6;  B: -3.4.3;  C: -2.5.9;  D: -2.5.8;  E: -2.4.5;  F: -2.3.2;
G: +1.2.1;  H: +2.3.3;  I: +2.4.6;  J: +2.5.8;  K: +2.5.10;  L: +2.6.12;  M: +2.6.13.
```

```
 | 1| 2|  3   |   4   | 5|  6   | 7|                                     k
 | 6| 9| 7   8| 2   3| 1| 4   5|10|   i                  ABCDEFGHIJKLM        1234567
  0|+I|+J +J|+M +M|+I|+L +L| F|    6   Beauty          0000010022022   1    0122120
     0|+I +I| F   F| F| E   E|+G|    9   Foundation      0000231030000   2    1020001
        | 0 +I| E   E| F|+H  D| F|    7   Splendor        0001220121000   3    1110010
           0| E   E| F| D  +H| F|    8   Victory         0001220121000   3    1110010
              | 0 +M|+I|+K +K| B|    2   Understanding   0100210010202   4    1001120
                 0|+I|+K +K| B|    3   Wisdom          0100210010202   4    1001120
                    0| C   C| B|    1   Crown           0120030030000   5    1002000
                       | 0 +K| A|    4   Severity        1011100100310   6    1012010
                          0| A|    5   Mercy           1011100100310   6    1012010
                             0|   10   Kingdom         2300031000000   7    0100000
```

The positions of (*understanding, wisdom*), (*severity, mercy*), (*beauty*) form a *5-clique* and positions (*beauty*), (*splendor, victory*), (*foundation*) form a *4-clique*. What mean the positions and cliques on the viewpoint of Kaballahh?

*

Since real ***communication networks*** are very large. Imagine here one a peculiar companionship ***Z*** consisting of *Adolf, Berta, Charles, Diana, Erik, Frieda, George, Helen, Ingvar and Jane.* They are mutually agreed that everyone communicates with the five, known to us, parlor companions. The latter circumstance had required of coordination, and someone had to do it.

Adolf – Berta, Charles, Diana, George, Jane;
Berta – Adolf, Charles, Helen, Ingvar, Jane;
Charles – Adolf, Berta, Diana, Erik, George;
Diana – Adolf, Charles, Erik, Frieda, Ingvar;
Erik – Charles, Diana, Frieda, Helen, Jane;
Frieda – Diana, Erik, George, Helen, Ingvar;
George – Adolf, Charles, Frieda, Helen, Jane;
Helen – Berta, Erik, Frieda, George, Ingvar;
Ingvar – Berta, Diana, Frieda, Helen, Jane;
Jane – Adolf, Berta, Erik, George, Ingvar.

This situation constitutes a *five-degree-regular structure* in which all the members seem to be in “equal position”.

Example 3.11. To present this situation make a corresponding structure model ***Z***:

```
A:-2.6.10;  B:-2.6.9;  C:-2.5.8;  D:-2.5.7;  E:-2.5.6;  F:-2.4.5;  G:-2.4.4;
            H:+2.3.3;  I:+2.4.5;  J:+2.5.7;  K:+3.10.25.
```

```
| 1| 2| 3| 4| 5| 6| 7| 8| 9|10|                  u_i
| F| A| D| H| C| B| I| J| E| G|  name     ABCDEFGHIJK   k
   0|-G| I| J|-D|-F| I|-E| I| H|  Frieda   00011111310   1
       0| H|-G| J| I|-D| I|-D| I|  Adolf    00020021310   2
           0|-C| I|-D| H|-E| I|-D|  Diana    00121002300   3
               0|-E| H| I|-B| H| H|  Helen    01101013110   4
                   0| H|-G|-A| H| H|  Charles  10011013110   5
                       0| I| I|-E|-A|  Berta    10011102300   6
                           0| H|-A|-D|  Ingvar   10020012300   7
                               0| K| H|  Jane     11002002201   8
                                   0|-B|  Erik     11011002201   9
                                       0|  George   11020004100  10
```

Unfortunately, the structure ***Z*** is *0-symmetric*, there do not “equality”, each member has its own private position. *Different position* means different connectivity, "relationships" with other members. Between ten members exists 11 different relationships, which is characterized by the binary signs (see frequency vectors $\boldsymbol{u_i}$). The problem lies here in the *grouping of strictly differentiated members*. This fact leads us back to the *sign structures* $\boldsymbol{GS_p}$. In selection of the sign must be proceeds from:

1) Selected sign must be exists *in case of each structural element*.
2) To keep in mind the *meaning of sign*, because the sign structure be formed on the aspect of sign.

In principle is the companionship decomposable to the eleven inseparable component sign structures $\boldsymbol{GS_p}$, and gives different groupings. This is inappropriate, and useful to go the other way.

Let to it is the rearranging the members by their “direct communication signs” ***HIJK*** of frequency-vectors.

Example 3.12. Rearranged by ***HIJK*** structure model ***Z***:

```
| 1| 2| 4| 5| 3| 6| 7| 8| 9|10|                       u_i
| F| A| H| C| D| B| I| J| E| G|    name      k       HIJK    R
   0|-G| J|-D| I|-F| I|-E| I| H|    Frieda    1       1310    1
       0|-G| J| H| I|-D| I|-D| I|    Adolf     2       1310    1
           0|-E|-C| H| I|-B| H| H|    Helen     4       3110    2
               0| I| H|-G|-A| H| H|    Charles   5       3110    2
                   0|-D| H|-E| I|-D|    Diana     3       2300    3
                       0| I| I|-E|-A|    Berta     6       2300    3
                           0| H|-A|-D|    Ingvar    7       2300    3
                               0| K| H|    Jane      8       2201    4
                                   0|-B|    Erik      9       2201    4
                                       0|    George   10       4100    5
```

The resulting grouping corresponds to the requirement of "direct communication signs", where the *ten positions **k*** reduces to *five groups*, with the members:

R_1= (*Frieda, Adolf*), R_2= (*Helen, Charles*), R_3= (*Diana, Berta, Ingvar*),
R_4= (*Jane, Erik*) and R_5= (*George*).

For finding the "similarity" of members can be use also *approximate or rounded-off* binary signs.

Example 3.13. Using the rounded-off binary signs:

Rounding-off: *a*= [*A:-2.6.10; B:-2.6.9*], *b*= [*C:-2.5.8; D:-2.5.7; E:-2.5.6*], *c*= [*F:-2.4.5; G:-2.4.4*], ***d*= [*H: +2.3.3; I:+2.4.5; J: +2.5.7*], *e*= (*K: +3.10.25*).**

Rounded binary signs: *a*:(*A, B*) ≈ –2.6, *b*:(*C, D, E*) ≈ –2.5, *c*:(*F, G*) ≈ –2.4, ***d*:(*H, I, J*) ≈ +2** ja ***e*: *K* ≈ +3.**

1		2	3			4		5			a	b	c	**d**	**e**		u^*_i	
F	**A**	**D**	**H**	**C**	**B**	**I**	**J**	**E**	**G**	***name***	*AB*	*CDE*	*FG*	***HIJ***	***K***	***k***	*abc**de***	***k****
0	*-G*	***I***	***J***	*-D*	*-F*	***I***	*-E*	***I***	***H***	**Frieda**	*00*	*011*	*11*	***131***	***0***	1	*022**50***	***1***
	0	***H***	*-G*	***J***	***I***	*-D*	***I***	*-D*	***I***	**Adolf**	*00*	*020*	*02*	***131***	***0***	2	*022**50***	***1***
		0	*-C*	***I***	*-D*	***H***	*-E*	***I***	*-D*	**Diana**	*00*	*121*	*00*	***230***	***0***	3	*030**50***	***2***
			0	*-E*	***H***	***I***	*-B*	***H***	***H***	**Helen**	*01*	*101*	*01*	***311***	***0***	4	*121**50***	***3***
				0	***H***	*-G*	*-A*	***H***	***H***	**Charles**	*10*	*011*	*01*	***311***	***0***	5	*121**50***	***3***
					0	***I***	***I***	*-E*	*-A*	**Berta**	*10*	*011*	*10*	***230***	***0***	6	*121**50***	***3***
						0	***H***	*-A*	*-D*	**Ingvar**	*10*	*020*	*01*	***230***	***0***	7	*121**50***	***3***
							0	***K***	***H***	**Jane**	*11*	*002*	*00*	***220***	***1***	8	*220**41***	***4***
								0	*-B*	**Erik**	*11*	*011*	*00*	***220***	***1***	9	*220**41***	***4***
									0	**George**	*11*	*020*	*00*	***410***	***0***	10	*220**50***	***5***

The resulting grouping by rounded-off binary signs:

k* $_1$= (*Frieda, Adolf*), ***k**** $_2$= (*Diana*), ***k**** $_3$= (*Helen, Charles, Berta, Ingvar*),
k* $_4$= (*Jane, Erik*) and ***k**** $_5$= (*George*).

We can see that there exist coincidences between the results of "direct communication signs" and rounding-off. The first way shall be considered as more distinct and therefore more reliable. The "rounding" of binary signs may prove to be quite arbitrary. Here can remark a *specific role of Jane and Erik* in this companionship, to their relationship ***K: +3.10.25*** includes all members and relationships, and they may be *coordinators.*

Such *0*-symmetric structures can be treats, investigate, and elements grouped in several ways:

1) By investigation of the selected sign structures GS_p.
2) By investigate on the basis of some selected binary signs formed the so-called complex sign structures.
3) By reordering the structural model by the given binary signs (example 3.12).
4) For reducing the positions to use the connected or "rounded" binary signs (example 3.13).

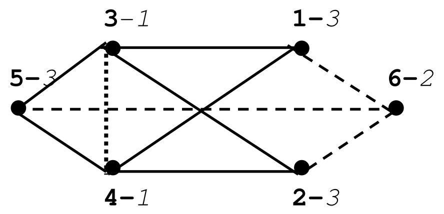
All of this requires a good knowledge of the subject and suitable choices the aspects for the investigation.

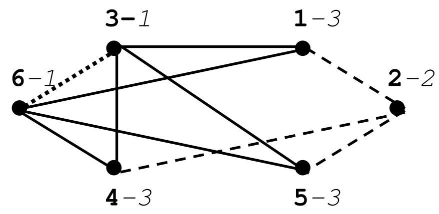
4. Structural equivalence and isomorphism

We demonstrate that isomorphic graphs have the same structure, which expressed in the form of structural equivalence of models, where:

1) Isomorphism is a one-to-one correspondence between elements, an isomorphic mapping from graph G_A to graph G_B is a bijection φ**:** $V_A \rightarrow V_B$:.
2) Isomorphism recognition does not recognize the structure, but the structure model recognize the structure with exactness up to isomorphism.
3) Structural equivalence is a coincidence or bijection on the level of binary signs, binary- and element positions.
4) Recognition the positions by binary signs is more simply than detecting the orbits on the ground of the group ***AutG***.

Example 4.1. Graphs G_A and G_B, their binary signs and structure models $\mathbf{SM}_A$ and $\mathbf{SM}_B$:





A:-2.5.7; B:-2.5.6;
C:+2.3.3; D:+2.5.7; E:+3.6.10.

A:-2.5.7; B:-2.5.6;
C:+2.3.3; D:+2.5.7; E:+3.6.10.

1	1	2	3	3	3		u_i	k	s_i
3	4	6	1	2	5	i	ABCDE		123
0	D	-B	C	C	C	3	01310	1	103
	0	-B	C	C	C	4	01310	1	103
		0	E	E	E	6	02003	2	003
			0	-A	-A	1	20201	3	210
				0	-A	2	20201	3	210
					0	5	20201	3	210

≈

1	1	2	3	3	3		u_i	k	s_i
3	6	2	1	4	5	i	ABCDE		123
0	D	-B	C	C	C	3	01310	1	103
	0	-B	C	C	C	6	01310	1	103
		0	E	E	E	2	02003	2	003
			0	-A	-A	1	20201	3	210
				0	-A	4	20201	3	210
					0	5	20201	3	210

Explanations:

a) Different graphs G_A and G_B have equivalent structure models $\mathbf{SM_A} \approx \mathbf{SM_B}$! This means that the structures are ***equivalent*** and the graphs ***isomorphic*** $G_A \cong G_B$.

b) The structural elements are divided to ***three positions*** *(equivalence classes, orbits)* ΩV_k and element pairs to ***five positions*** ΩR_n, where the adjacent elements or "edges" divided to *three binary(+)positions (full line, a dotted, dashed-line)* that coincides with binary signs ***C, D, E*** correspondingly.

c) The column u_i of model consists of the ***frequency vectors***, which for the element i show its relations with other elements. On the basis of vectors u_i are arranged the positions in model.

d) The column s_i of model consists of the ***position vectors*** that represent the connections of element i with elements in corresponding positions k. If on the framework of frequency vectors arises differences of position vectors, then by lasts does a complementary partition into classes.

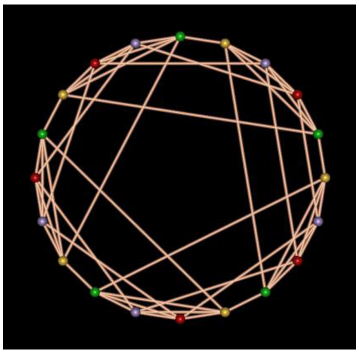
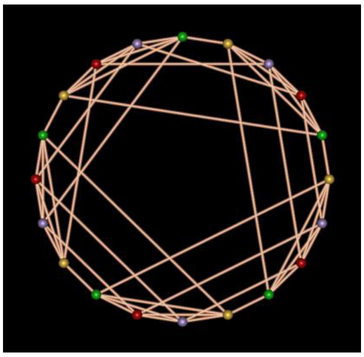
Here it may be noted that the first primitive "distance matrix" was presented already in 1973 by S. Toida [40], as isomorphism identification attribute. Indeed, the distance matrix can detect the isomorphism or "non-isomorphism" for quite many graphs, but it is by no means reliable.

The structure model **SM** is a *canonical description* of structure (graph) with exactness up to *binary signs, positions, isomorphism and others* structural attributes. The problem of *canonical representation of the graphs* was set by Lazlo Babai in 1977th [1]. The presentation ways are proposed much [5, 9]. Unfortunately, they do not contain information about the structure.

If the structure models of graphs $\boldsymbol{G}$ and $\boldsymbol{H}$ are *equivalent* $\mathbf{SM}_G \approx \mathbf{SM}_H$ then the graphs are *isomorphic* $\boldsymbol{G} \cong \boldsymbol{H}$.

The isomorphism problem is to design an algorithm that recognizes the isomorphism of two objects. The ***graph isomorphism problem*** came into prominence in 1857, when Arthur Cayley reported his research on organic isomers [3]. Two graphs called isomorphic, if they differ only in the labeling of their vertices.

Example 4.2. Graphs $\boldsymbol{Pra_A}$ and $\boldsymbol{Pra_B}$, designed especially for testing the structural equivalence of "very similar" poly-symmetric graphs that have common basic, but different perfected binary signs:

Common basic binary signs of $\boldsymbol{Pra_A}$ and $\boldsymbol{Pra_B}$:

A:-3.8.10; B:-3.6.7; C:-2.4.4; D:-2.3.2; ***E:+2.4.6****;* ***F:+3.8.16.***

Perfected by matrix product $\boldsymbol{E}^{n=5}$ binary signs and structure model **SM** of graph $\boldsymbol{Pra_A}$:

Marking the basic binary signs	0	-A	-B	-C		-D	**E**		**F**
Productive binary signs $\boldsymbol{e}^5$	180	125	110	165	160	80	231	233	210
Perfected binary signs	0	-A	-B	-C1	-C2	-D	**E1**	**E2**	**F**
Frequency vector	–	2	4	4	2	3	**2**	**1**	**2**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	i	u_i ABCCD**EEF**	k
0	**E2**	**E1**	**E1**	**F**	C2	C1	C1	**F**	C2	C1	C1	D	A	B	B	D	A	B	B	**1**	24422**212**	1
	0	**E1**	**E1**	C2	**F**	C1	C1	C2	**F**	C1	C1	A	D	B	B	A	D	B	B	**2**	24422**212**	1
		0	**E2**	C1	C1	**F**	C2	C1	C1	**F**	C2	B	B	D	A	B	B	D	A	**3**	24422**212**	1
			0	C1	C1	C2	**F**	C1	C1	C2	**F**	B	B	A	D	B	B	A	D	**4**	24422**212**	1
				0	**E2**	**E1**	**E1**	D	A	B	B	**F**	C2	C1	C1	A	D	B	B	**5**	24422**212**	1
					0	**E1**	**E1**	A	D	B	B	C2	**F**	C1	C1	D	A	B	B	**6**	24222**212**	1
						0	**E2**	B	B	D	A	C1	C1	**F**	C2	B	B	A	D	**7**	24222**212**	1
							0	B	B	A	D	C1	C1	C2	**F**	B	B	D	A	**8**	24222**212**	1
								0	**E2**	**E1**	**E1**	A	D	B	B	**F**	C2	C1	C1	**9**	24222**212**	1
									0	**E1**	**E1**	D	A	B	B	C2	**F**	C1	C1	**10**	24222**212**	1
										0	**E2**	B	B	A	D	C1	C1	**F**	C2	**11**	24222**212**	1
											0	B	B	D	A	C1	C1	C2	**F**	**12**	24222**212**	1
												0	**E2**	**E1**	**E1**	C2	**F**	C1	C1	**13**	24222**212**	1
													0	**E1**	**E1**	**F**	C2	C1	C1	**14**	24222**212**	1
														0	**E2**	C1	C1	C2	**F**	**15**	24222**212**	1
															0	C1	C1	**F**	C2	**16**	24222**212**	1
																0	**E2**	**E1**	**E1**	**17**	24222**212**	1
																	0	**E1**	**E1**	**18**	24222**212**	1
																		0	**E2**	**19**	24222**212**	1
																			0	**20**	24222**212**	1

Perfected by matrix product $\boldsymbol{E}^{n=7}$ binary signs and structure model **SM** of graph $\boldsymbol{Pra_B}$:

Basic binary signs	0	-A	-B		-C			-D	**E**		**F**
Productive signs $\boldsymbol{e}^7$	4410	3437	3276	3277	4081	4088	4011	3010	4831	4803	4445
Perfected signs	0	-A	-B1	-B2	-C1	-C2	-C3	-D	**E1**	**E2**	**F**
Frequency vector	–	2	2	2	2	2	2	2	**2**	**1**	**2**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	*i*	u_i ABBCCCD**EEF**	*k*
0	**E1**	**E2**	**E1**	**F**	C1	C2	C3	**F**	C3	C2	C1	D	B2	B1	A	D	A	B1	B2	1	2222222**212**	1
	0	**E1**	**E2**	C3	**F**	C1	C2	C1	**F**	C3	C2	A	D	B2	B1	B2	D	A	B1	2	2222222**212**	1
		0	**E1**	C2	C3	**F**	C1	C2	C1	**F**	C3	B1	A	D	B2	B1	B2	D	A	3	2222222**212**	1
			0	C1	C2	C3	**F**	C3	C2	C1	**F**	B2	B1	A	D	A	B1	B2	D	4	2222222**212**	1
				0	**E1**	**E2**	**E1**	D	A	B1	B2	**F**	C1	C2	C3	A	D	B2	B1	5	2222222**212**	1
					0	**E1**	**E2**	B2	D	A	B1	C3	**F**	C1	C2	B1	A	D	B2	6	2222222**212**	1
						0	**E1**	B1	B2	D	A	C2	C3	**F**	C1	B2	B1	A	D	7	2222222**212**	1
							0	A	B1	B2	D	C1	C2	C3	**F**	D	B2	B1	A	8	2222222**212**	1
								0	**E1**	**E2**	**E1**	A	B1	B2	D	**F**	C3	C2	C1	9	2222222**212**	1
									0	**E1**	**E2**	D	A	B1	B2	C1	**F**	C3	C2	10	2222222**212**	1
										0	**E1**	B2	D	A	B1	C2	C1	**F**	C3	11	2222222**212**	1
											0	B1	B2	D	A	C3	C2	C1	**F**	12	2222222**212**	1
												0	**E1**	**E2**	**E1**	C3	**F**	C1	C2	13	2222222**212**	1
													0	**E1**	**E2**	C2	C3	**F**	C1	14	2222222**212**	1
														0	**E1**	C1	C2	C3	**F**	15	2222222**212**	1
															0	**F**	C1	C2	C3	16	2222222**212**	1
																0	**E1**	**E2**	**E1**	17	2222222**212**	1
																	0	**E1**	**E2**	18	2222222**212**	1
																		0	**E1**	19	2222222**212**	1
																			0	20	2222222**212**	1

Explanations:

- **a)** The structure models of Pra_A and Pra_B are ***non equivalent,*** it is recognized already with the difference of frequency vectors, and graphs are ***non isomorphic.***
- **b)** Graph Pra_A has *five* binary(−)positions by *–A, −C2,* and *−D* with power 20 and two binary positions by *−B* and *−C1* with power 40.
- **c)** Graph Pra_B has *seven* binary(−)positions whit power 20.
- **d)** Both graphs have *three* binary(+)positions ***E1, E2*** and ***F*** with power 20.

For recognition the equivalence of structure models SM_A and SM_B is necessary and sufficient:

1) Detecting the coincidence of the *sequences of binary signs* $\{\pm d.n.q._{ij}\}_A$ and $\{\pm d.n.q._{ij}\}_B$;
2) Detecting the coincidence of the *frequency vectors* $\{u_i\}_A$ and $\{u_i\}_B$;
3) Detecting the coincidence of the *position vectors* $\{s_i\}_A$ and $\{s_i\}_B$.

*

It is possible to construct such *bisymmetric and strongly regular graphs* that have very small binary graphs in case of large number of vertices. We call these ***strongly symmetric graphs.*** Look to constructed by M. Nechepurenko, M. Klin et al *strongly symmetric graphs* Sib_A and Sib_B with 40 vertices [13]. These graphs have ***common binary signs***: *–A:–2.6.8* (complement has ***+B:+2.20.142***) and ***+B:+2.4.6*** (the complement has *–A:–2.20.144*). From binary signs conclude that Sib_A and Sib_B are *4-clique-, 2-distance-* and *12-degree regular.* From coincidence the binary signs of Sib_A and Sib_B conclude the *coincidence of the symmetry properties.*

As in case of strongly symmetric graphs the product identification no works, we must use another perfection ways. The *high (second) degree pair signs* (see introduction 1.1) of Sib_A and Sib_B are $-A^{m=2}=-3.18.48$ and $+B^{m=2}=+3.20.64$, and anew coincide. A binary graph of third degree $g_{ij}^{m=3}$ no arise, it is empty ∅.

Now must to form for *second degree binary graphs* $g_{ij}^{m=2}$ of Sib_A and Sib_B with help the *local structure models* $SM_{ij}^{m=2}$ (see introduction 1.2). For this we open in both graphs the binary graph $g_{ij}^{m=2}{}_A$ and $g_{ij}^{m=2}{}_B$, such that correspond to pair sign $+B^{m=2}$.

Example 4.3. Binary signs of second degree binary graphs $g_{ij}^{m=2}$ of Sib_A and Sib_B are:

Binary signs of second degree binary graph $g_{ij}^{m=2} \subset Sib_A$ in local structure model $SM_{ij}^{m=2}{}_A$:

–A=–2.6.8; –B=–2.4.4; –C=–2.3.2; ***D=+2.4.6; E=+3.12.28; F=+3.20.46.***

Binary signs of second degree binary graph $g_{ij}^{m=2} \subset Sib_B$ in local structure model $SM_{ij}^{m=2}{}_B$:

–A=–2.6.8; –B=–2.4.4; ***C=+2.4.6; D=+3.12.24; E=+3.20.46.***

Explanations:

a) From *differences* of binary signs of second degree binary graphs signs conclude *non-equivalence of local structure models*, $\mathbf{SM}_{ij}{}^{m=2}{}_{A} \neq \mathbf{SM}_{ij}{}^{m=2}{}_{B}$.

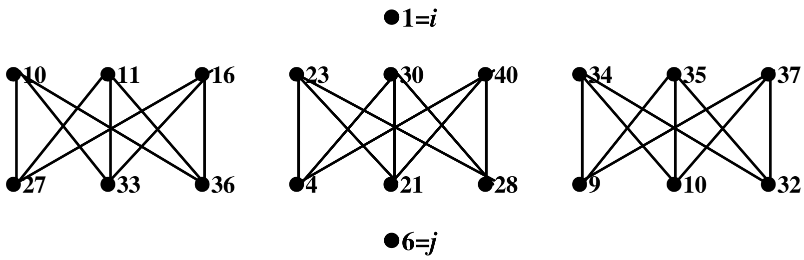
b) From non-equivalence of local structure models conclude *non-equivalence of structures* $\boldsymbol{Sib_A}$ and $\boldsymbol{Sib_B}$.

Thus, from non-isomorphism of binary graphs $\boldsymbol{g_{ij}{}^{m}{}_{A}}$ and $\boldsymbol{g_{ij}{}^{m}{}_{B}}$ of corresponding strongly symmetric graphs $\boldsymbol{G_A}$ and $\boldsymbol{G_B}$ concludes ***structural non-equivalence*** and ***non-isomorphism*** of $\boldsymbol{G_A}$ and $\boldsymbol{G_B}$.

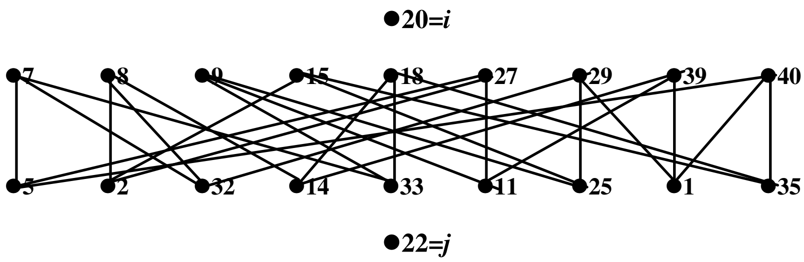
For showing the differences of $\boldsymbol{Sib_A}$ and $\boldsymbol{Sib_B}$ we demostrate the kernels of second degree pair graphs.

Example 4.4. The kernels of second degree pair graphs of very similar structures $\boldsymbol{Sib_A}$ and $\boldsymbol{Sib_B}$:

Kernel of $\boldsymbol{g_{1\text{-}6}{}^{m=2}} \subset \boldsymbol{Sib_A}$:



Kernel of $\boldsymbol{g_{20\text{-}22}{}^{m=2}} \subset \boldsymbol{Sib_B}$:

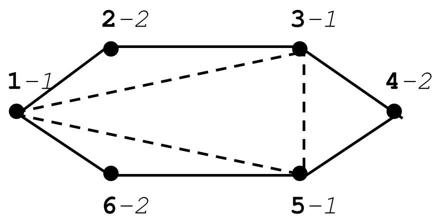


Explanation: There is no doubt that the bipartite kernels are different.

5. Adjacent structures and reconstruction problem

Example 5.1. Partially symmetric structure *GS.37(6.9.4)* [28, 35] with two element positions and four binary positions (two binary(+)- and two binary(–)positions, its graph, structure model, characteristics of changes and morphisms:

A:-2.4.5; B:-2.3.2;
C:+2.3.3; D:+2.4.5.



1	1	1	2	2	2		u_i	k	s_i
1	**3**	**5**	**2**	**4**	**6**	*i*	*ABCD*		12
0	**D**	**D**	**C**	-A	**C**	1	1022	1	22
	0	**D**	**C**	**C**	-A	3	1022	1	22
		0	-A	**C**	**C**	5	1022	1	22
			0	-B	-B	2	1220	2	20
				0	-B	4	1220	2	20
					0	6	1220	2	20

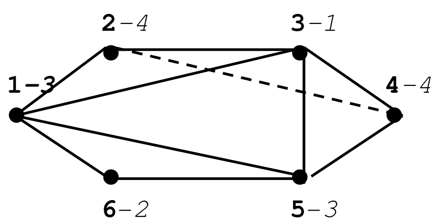
	GS^{adj}_n	1	2
	GS^{sup}_{n-}	29	30
GS.37	k.k' (p)	2.2 (-B)	1.2 (-A)
	PF^{sup}_{n-}	3/6	3/6
	GS^{sub}_{n+}	72	76
GS.37	k.k' (p)	1.1 (+D)	1.2 (+C)
	PF^{sub}_{n+}	3/9	6/9

Explanations:

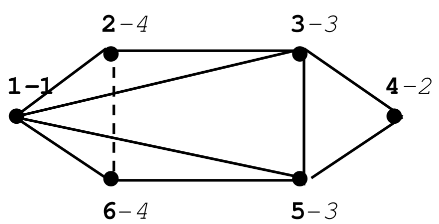
- **a)** GS^{sup}_{n-} and GS^{sub}_{n+} denotes the *ordering numbers* of adjacent superstructures and adjacent substructures in the system of structures with six elements [28, 35];
- **b)** ***k,k'*** – index of partial model $\mathbf{SM}_{k,k'}$, whither belong the binary position ***(p)***;
- **c)** PF_n – *morphism probability*.

Example 5.2. Three *isomorphic graphs* that represent the ***adjacent superstructure*** $GS^{sup}_{n=-B}$, (*GS.29)* [28, 35] of structure *GS.37* (example 5.1). These are obtained by *adding* the connections 2-4, 2-6 and 4-6 (dashed line) to binary(–)position *–B* of *GS.37.* Their *common binary signs* and *equivalent structure models* $\mathbf{SM_1} \equiv \mathbf{SM_2} \equiv \mathbf{SM_3}$:

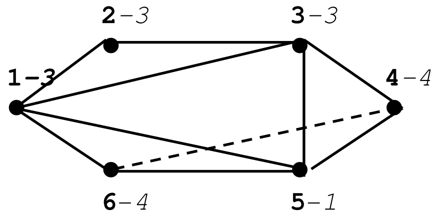
A:-2.5.8; B:-2.4.5; C;-2.3.2;
D:+2.3.3; E:+2.4.5.



1	2	3	3	4	4		u_i		s_i
3	**6**	**1**	**5**	**2**	**4**	*i*	*ABCDE*	*k*	1234
0	-B	**E**	**E**	**E**	**E**	3	01004	1	0022
	0	**D**	**D**	-C	-C	6	01220	2	0020
		0	**E**	**D**	-A	1	10022	3	1111
			0	-A	**D**	5	10022	3	1111
				0	**D***	2	10121	4	1011
					0	4	10121	4	1011



1	2	3	3	4	4		u_i		s_i
1	**4**	**3**	**5**	**2**	**6**	*i*	*ABCDE*	*k*	1234
0	-B	**E**	**E**	**E**	**E**	1	01004	1	0022
	0	**D**	**D**	-C	-C	4	01220	2	0020
		0	**E**	**D**	-A	3	10022	3	1111
			0	-A	**D**	5	10022	3	1111
				0	**D***	2	10121	4	1011
					0	6	10121	4	1011

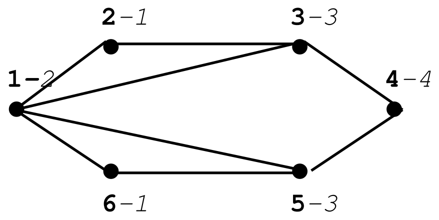


1	2	3	3	4	4		u_i		s_i
5	**2**	**1**	**3**	**4**	**6**	***i***	*ABC**DE***	***k***	1234
0	**−**B	***E***	***E***	***E***	***E***	**5**	010**04**	1	00**22**
	0	***D***	***D***	−C	−C	**2**	012**20**	2	00**2**0
		0	***E***	−A	***D***	**1**	100**22**	3	**1111**
			0	***D***	−A	**3**	100**22**	3	**1111**
				0	***D****	**4**	101**21**	4	**1**0**11**
					0	**6**	101**21**	4	**1**0**11**

Explanation: Equivalent structure models differ from each other only numbered elements in different positions in the division.

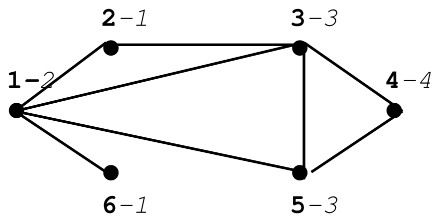
<u>Example 5.3.</u> The different ***adjacent substructures*** $\boldsymbol{GS^{sub}_{n=+D}}$, (*GS.72)* [28, 35] and $\boldsymbol{GS^{sub}_{n=+C}}$, (*GS.76*) of structure *GS.37* (example 5.1) that obtained by *removing* the connection 3-5 from binary(+)position ***+D*** and *removing* the connection 5-6 from binary(+)position ***+C*** correspondingly. Their *non-isomorphic graphs, different binary signs* and *non-equivalent structure models* $\mathbf{SM}_A$ and $\mathbf{SM}_B$:

A:−2.4.4; B:−2.3.2;
C:+2.3.3; D:+3.4.4.



1	1	2	3	3	4		u_i		s_i
2	**6**	**1**	**3**	**5**	**4**	***i***	*AB**CD***	***k***	1234
0	−B	***C***	***C***	−B	−B	**2**	03**20**	1	0**11**0
	0	***C***	−B	***C***	**−**B	**6**	03**20**	1	0**11**0
		0	***C***	***C***	−A	**1**	10**40**	2	**2**0**2**0
			0	−A*	***D***	**3**	11**21**	3	**11**0**1**
				0	***D***	**5**	11**21**	3	**11**0**1**
					0	**4**	12**02**	4	00**2**0

A:−3.5.6; B:−2.4.5; C:−2.3.2;
D:+1.2.1; E:+2.3.3; F:+2.4.5.



1	2	3	4	5	6		u_i		s_i
3	**1**	**5**	**2**	**6**	**4**	***i***	*ABC**DEF***	***k***	123456
0	***F***	***F***	***E***	−C	***E***	**3**	001**022**	1	0**111**0**1**
	0	***E***	***E***	***D***	**−**B	**1**	010**121**	2	10**111**0
		0	−B	−C	***E***	**5**	011**021**	3	**11**000**1**
			0	C*	−C	**2**	012**020**	4	**11**0000
				0	−A	**6**	103**100**	5	0**1**0000
					0	**4**	111**020**	6	**1**0**1**000

Explanation: By different binary positions obtained adjacent structures are *non-equivalent.*

Each structure *GS* is an adjacent substructure GS^{sub}_n or adjacent superstructure G^{sup}_n of some other structures.

Morphism ***F*** is ***reversible –*** in each adjacent structure $\boldsymbol{GS^{adj}}$ of ***GS*** exist an "*reverse position*" $\boldsymbol{\Omega R^{rev}}$, whereat used *reverse morphism* $\boldsymbol{F^{rev}}$ ***reconstruct*** the initial structure ***GS***, $\boldsymbol{F^{rev}: GS^{adj} \to GS}$.

Let the structure on example 5.2 is an initial structure ***GS*** that has an adjacent substructure $\boldsymbol{GS^{sub}_n}$ in the forms of structure on example 5.1. Then ***GS*** can be *reconstruct* by adding a connection to the *reverse position –B* of $\boldsymbol{GS^{sub}_n}$ with morphism probability $\boldsymbol{PF^{rev}}$=3/6.

The reversing of morphism is valid both in the case of adjacent sub- $\boldsymbol{GS^{sub}_{n+}}$ and super-structures $\boldsymbol{Gs^{sup}_{n-}}$. Indeed, structure ***GS*** can be reconstructed by each of its adjacent structure $\boldsymbol{GS^{adj}}$ separately. On the set $\{\boldsymbol{GS^{adj}_n}\}$ of all the adjacents of $\boldsymbol{GS}$ there exists a certain *set of reverse morphisms* $\{\boldsymbol{F^{rev}_n}\}$, $n\in[1, N]$, such that each its disjunctive element $(\boldsymbol{F^{rev}_1: GS^{adj}_1\to GS})\vee\ldots\vee(\boldsymbol{F^{rev}_N: GS^{adj}_N\to GS})$ *reconstructs the structure* $\boldsymbol{GS}$ *separately.*

Thus, to be precise, the morphisms exist between the binary positions of structures.

If morphisms $\boldsymbol{F_n: GS\to GS^{adj}_n}$ are applied to binary positions $\boldsymbol{\Omega R_1,\ldots,\Omega R_n,\ldots,\Omega R_N}$ of $\boldsymbol{GS}$ disjunctivelly, $\boldsymbol{F_1\vee\ldots\vee F_n\vee\ldots\vee F_N}$, then $\boldsymbol{GS}$ is ***decomposed*** *(deconstructed)* to its *adjacent structures* $\boldsymbol{GS^{adj}_1,\ldots,GS^{adj}_n,\ldots,GS^{adj}_N}$.

Non-decomposable structures do not exist.

If structure $\boldsymbol{GS}$ is ***decomposed*** *(deconstructed)* to its ***adjacent substructures*** $\boldsymbol{GS^{sub}_1,\ldots,GS^{sub}_n,\ldots,GS^{sub}_N}$, then their ***union*** $\cup(\boldsymbol{GS\backslash e_{ij}})_n$, $n^+\in[1, N^+]$, ***reconstruct*** *(recompose)* the structure $\boldsymbol{GS}$.

This applies in particular for union of adjacency matrices $\cup(\boldsymbol{E\backslash e_{ij}})_n = \boldsymbol{E}$.

If structure $\boldsymbol{GS}$ is ***decomposed*** *(deconstructed)* toi ts ***adjacent superstructures*** $\boldsymbol{GS^{sup}_1,\ldots,GS^{sup}_n,\ldots,GS^{sup}_N}$, then their ***intersection*** $\cap(\boldsymbol{GS\cup e_{ij}})_n$, $n^-\in[1, N^-]$, ***reconstruct*** *(recompose)* the structure $\boldsymbol{GS}$.

Thus, the ***reconstructing (restoring) of structure is inevitable***, non-reconstructive structures do not exist.

The *reconstruction problem* is known as ***Ulam's Conjecture*** and reflects the isomorphism relations between two graphs and their $(\boldsymbol{G\backslash v_i})$-subgraphs [42]. It is formulated as follows: "Let graph $\boldsymbol{G}$ has $\boldsymbol{p}\geq 3$ vertices $\boldsymbol{v_i}$ and $\boldsymbol{H}$ has $\boldsymbol{p}\geq 3$ vertices $\boldsymbol{u_i}$. If for each $\boldsymbol{i}$, the sub-graphs $\boldsymbol{G_i=G\backslash v_i}$ and $\boldsymbol{H_i=H\backslash u_i}$ are isomorphic, then the graphs $\boldsymbol{G}$ and $\boldsymbol{H}$ are isomorphic".

This problem has been over the past half century, one of under active consideration graph theoretical problem, but the ultimate solutions have only some graph classes. Why so? Evidently be interested on the question**:** contain the collection of sub-graphs $\boldsymbol{G\backslash v_i}$ of $\boldsymbol{G}$ enough information about graph $\boldsymbol{G}$ itself? On the structural aspect is the *procedure* of this conjecture *nonsense,* because, if given graphs $\boldsymbol{G}$ and $\boldsymbol{H}$ then on the ground of *structure models* $\mathbf{SM}_{\boldsymbol{G}}$ and $\mathbf{SM}_{\boldsymbol{H}}$ we obtain the complete information about their isomorphism and isomorphism of their adjacent graphs. Other ways are for us here senseless.

Ulam's Conjecture treats the reconstruction on the aspect of removing of the vertices, but we treat it on the aspect of adding and removing of edges. This not changes the essence of reconstruction, because all remains to the frame of graphs (structures) and their adjacent graphs (-structures), i.e. in our case to the frame of *morphisms* $\boldsymbol{F_n}$*.* Already old master W. T. Tutte emphasized that reconstruction-problem must be solve on the basis of isomorphism classes, that we also have followed [41].

By help of the morphisms between adjacent structures are generated the ***system of structures*** with five elements [32] (where 72 morphisms connect 34 structures) and the system with six elements [35] (where 572 morphisms connect 156 structures). Principally it can be generated for all the structures. It also shows the inevitability of reconstructing.

The first sample of non-isomorphic graphs with up to six vertices was represented by Frank Harary in 1969th [7]. Later, F. Harary and E. Palmer had calculated the number of non-isomorphic graphs (i.e. structures) up to 24 vertices [8]. R. Read and F. Wilson have in "Graph Atlas" also given the diagrams of graphs up to seven vertices [16]. But so far do not are discussed about the relationships between adjacent graphs, i.e. morphisms. It is not much possible that someone would have tried to do anything like on the base of combinatorics, algebra or other classical attributes.

Conclusion

Here was demonstrated that the nature of the structure is revealed on the base of the *relationships between the elements* and their *positions* [37]. It is presentable in the form of *structure's model*.

Significant is this, that the different problems, such as recognition of structure, detecting of structural positions (orbits), isomorphism, reconstructions and the systems of adjacent structures (morphisms) are treatable *on the base of the same attribute – on the structure's model.*

Recognition of the structure can be characterized by the following sequence of attributes: *adjacency matrix→ identification of the element pairs→ binary signs→ decomposition→ structural positions→ structure's model*.

Recognition of the structural properties is based on the structural model and is realizable on three directions:

1) *Structure's model→ structural properties → sign- and position structures*;
2) *Structure's model→ equivalence of structural models→ isomorphism*;
3) *Structure's model→ elementary structural changes→ adjacent structures→ reconstruction problem → system of structures*.

The structural models open some hidden sides of the graphs that complement our knowledge about graphs. It is expressively sees in case of known graphs.

There exists an agreement that the structure is an inseparable attribute of all the really existing objects. Structure exists there where the relations between element pairs are recognizable. The relations are simple presentable in case of chemical compounds, genetic formations and some networks. In case of ecological and social communities must be previously to agree on the *aspect* of decomposition the object to its elements and their connections (relations). If be accepted the existence of structure, then is desirable accept also their attributes. For example, accept the positions and the relationships between these.

Presumably, that such attributes for chemists, biologists and others are unaccustomed phenomena, but it is the *structural reality* and worth thinking about it. It is also clear that the structure of natural objects not easily recognizable.

Hidden remains the problem of multiplication of the adjacent matrices. On this base has been developed a spectral treatment of the graphs, *spectral graph theory*. But we are interested in the problem of high degree adjacent matrices $\boldsymbol{E}^n$. A practical meaning has the following fact: 1) if to multiply the adjacency matrices $\boldsymbol{E}$, then enlarge the values of the elements as well as the number of different values; 2) the enlargement takes place only to a certain degree $\boldsymbol{n}$, after which the enlarging stopped; 3) remains the question: what for the values of vertex pairs in the obtained matrix $\boldsymbol{E}^n$ detect the binary positions, including on this base obtained vertex positions?

In principle, the structure's model can be based only on the elements of multiplied adjacent matrices, if would known the meaning of those elements. It is not known what represent the elements of obtained matrices, and to what degree must the matrices to multiply. There is only alluded that these elements characterize the longest paths between the vertices. This is doubtful, since these also appear in the main diagonal, while the relationships between the vertices, occasionally turn out to be zero. In present case we cannot distinguish from each other even adjacent and non-adjacent pairs of elements. Obviously, this nobody not interested. Already in 1976 were drawn attentions to the too one-sided approach to the graphs that impede the development of graph theory [12].

The preliminary binary signs are indispensable (required), the more that in case of strongly regular graphs, the multiplication of adjacency matrices works only partially.

Hope, that this paper gives a sufficient overview about the nature of structure

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